7/8 Mathematics A

Parent and student support booklet

This booklet is a reference guide of information and worked examples. Not all sections may be relevant to the booklets you have.

If you require further support I can be emailed Melissa.eagles@det.nsw.edu.au or contact during school hours on 67851184.

Sections
Addition
Subtraction
Multiplication – what is? Works both ways, multiples, short and long multiplication examples
Division – how it works, short and long division, carrying numbers
Units of measurement – basic units, compound measures, speed, density
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Measurement conversion – $mm^2/cm^2/m^2/km^2$
Measurement Perimeter – explanation and examples
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Measurement Triangles (area) - formula and explanation of what meant by height
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Circles – properties, terminology, parts of a circle
Circle – Circumference and diameter
Area of a circle – finding the area using the diameter or radius
Mathematical signs and symbols
Prime numbers, multiplication table, squares, cubes and roots
Shapes
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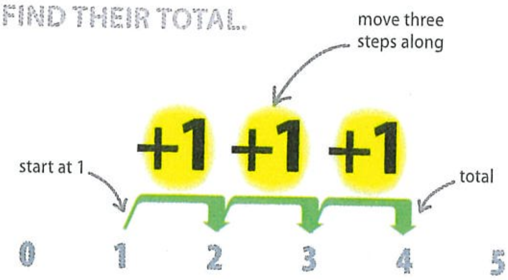
Addition

NUMBERS ARE ADDED TOGETHER TO FIND THEIR TOTAL. THIS RESULT IS CALLED THE SUM.

SEE ALSO
Subtraction 17 ❯
Positive and negative numbers 34–35 ❯

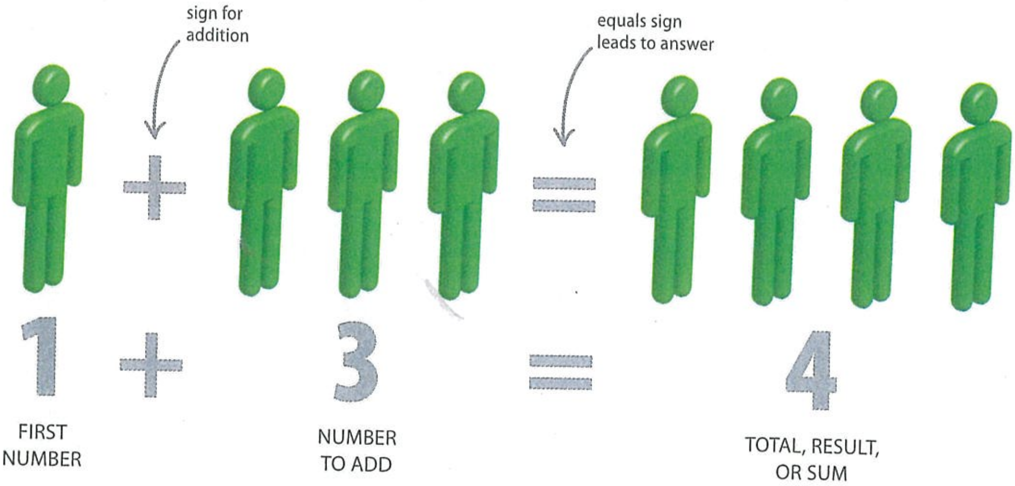
Adding up

An easy way to work out the sum of two numbers is a number line. It is a group of numbers arranged in a straight line that makes it possible to count up or down. In this number line, 3 is added to 1.



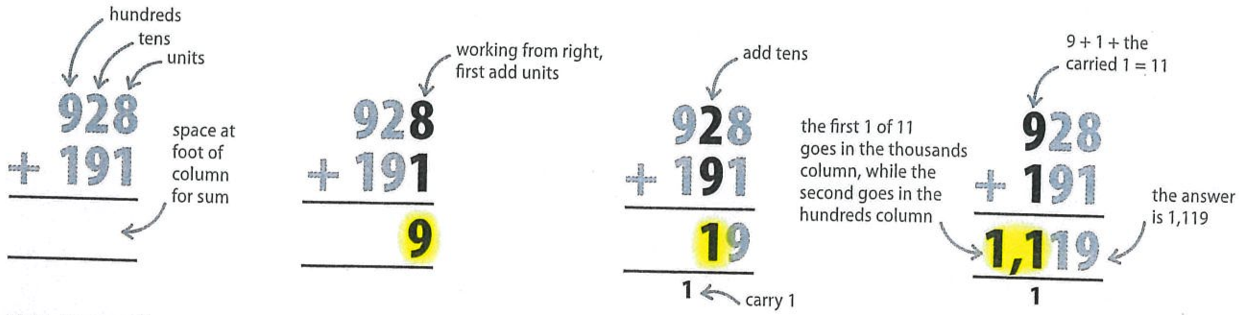
◁ **Use a number line**
To add 3 to 1, start at 1 and move along the line three times – first to 2, then to 3, then to 4, which is the answer.

▷ **What it means**
The result of adding 3 to the start number of 1 is 4. This means that the sum of 1 and 3 is 4.



Adding large numbers

Numbers that have two or more digits are added in vertical columns. First, add the units, then the tens, the hundreds, and so on. The sum of each column is written beneath it. If the sum has two digits, the first is carried to the next column.



First, the numbers are written with their units, tens, and hundreds directly above each other.

Next, add the units 1 and 8 and write their sum of 9 in the space underneath the units column.

As the sum of the tens has two digits, write the second underneath and carry the first to the next column.

Then add the hundreds and the carried digit. As this sum has two digits, the first goes in the thousands column.

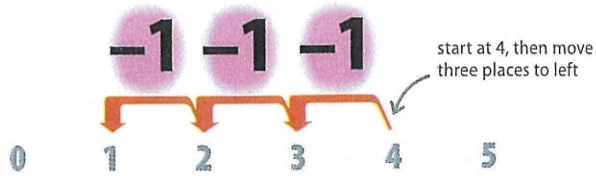
− Subtraction

A NUMBER IS SUBTRACTED FROM ANOTHER NUMBER TO FIND WHAT IS LEFT. THIS IS KNOWN AS THE DIFFERENCE.

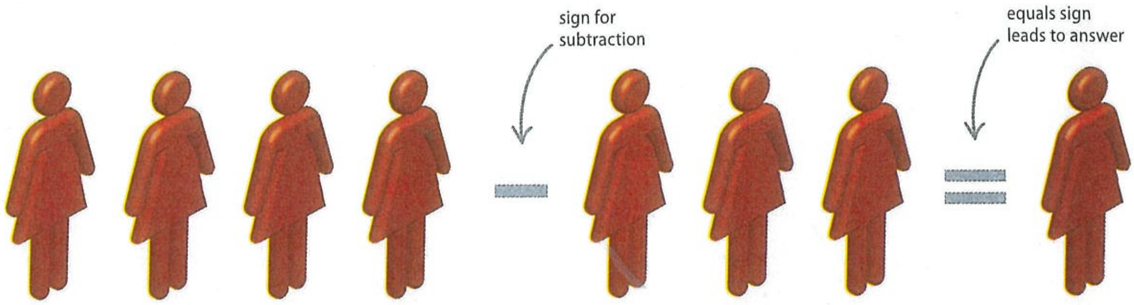
SEE ALSO
❮ 16 Addition
Positive and negative numbers 34–35 ❯

Taking away

A number line can also be used to show how to subtract numbers. From the first number, move back along the line the number of places shown by the second number. Here 3 is taken from 4.



◁ **Use a number line** To subtract 3 from 4, start at 4 and move three places along the number line, first to 3, then 2, and then to 1.



▷ **What it means** The result of subtracting 3 from 4 is 1, so the difference between 3 and 4 is 1.

4 − 3 = 1

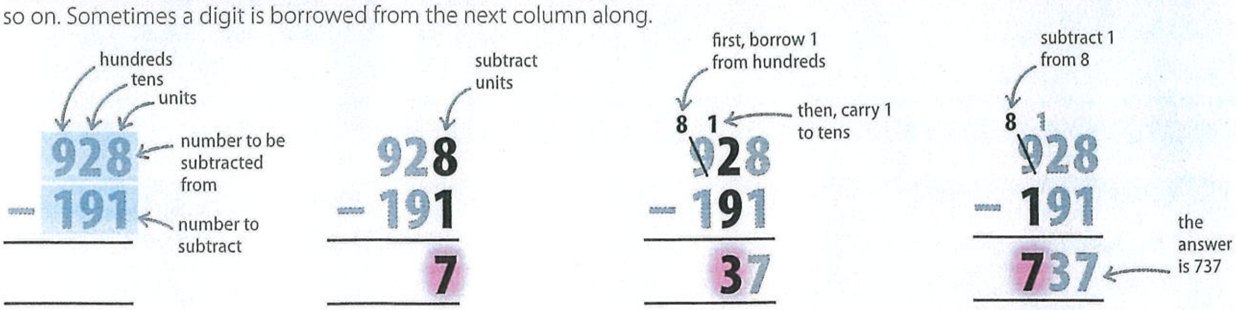
FIRST NUMBER

NUMBER TO SUBTRACT

RESULT OR DIFFERENCE

Subtracting large numbers

Subtracting numbers of two or more digits is done in vertical columns. First subtract the units, then the tens, the hundreds, and so on. Sometimes a digit is borrowed from the next column along.



First, the numbers are written with their units, tens, and hundreds directly above each other.

Next, subtract the unit 1 from 8, and write their difference of 7 in the space underneath them.

In the tens, 9 cannot be subtracted from 2, so 1 is borrowed from the hundreds, turning 9 into 8 and 2 into 12.

In the hundreds column, 1 is subtracted from the new, now lower number of 8.

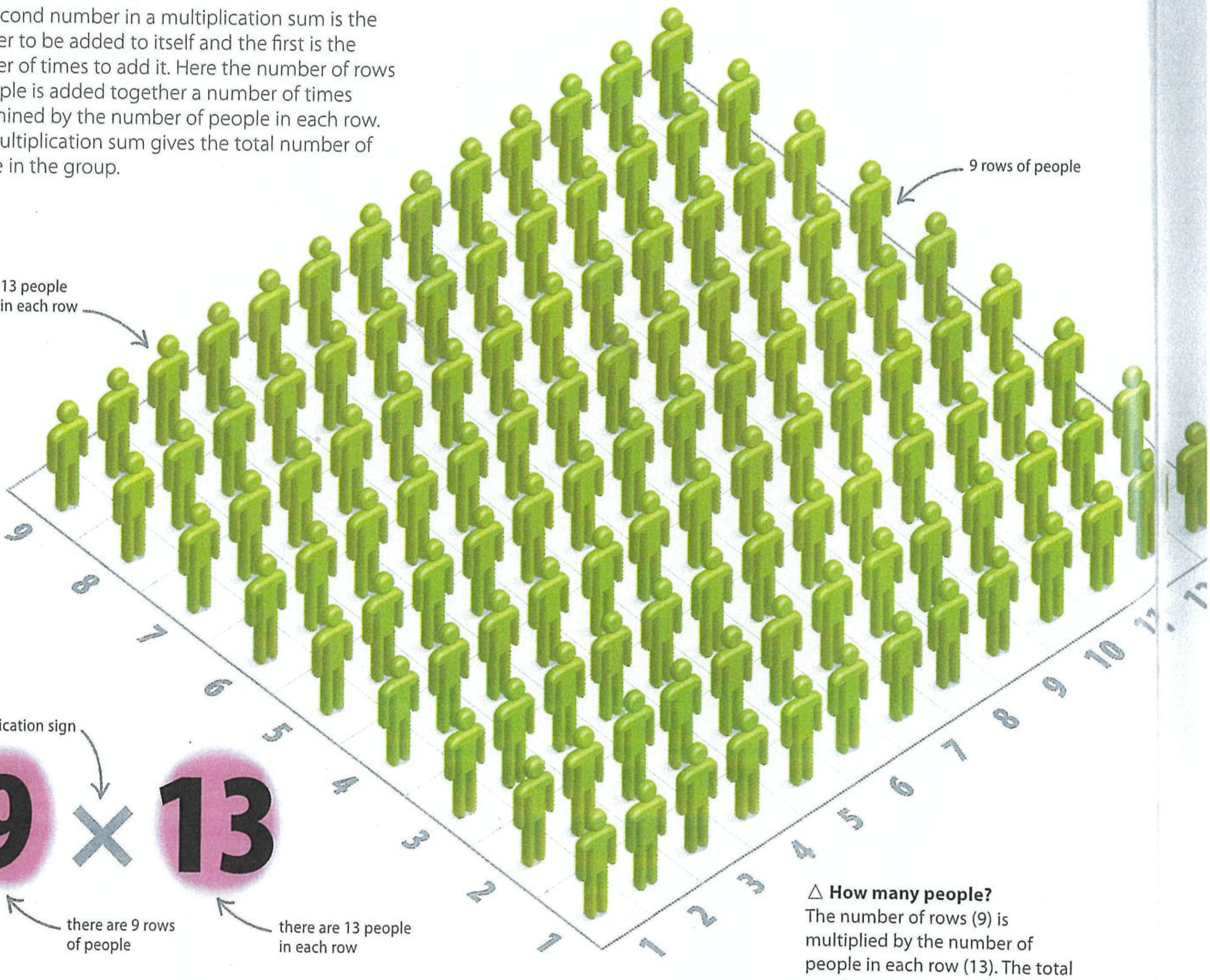
Multiplication

SEE ALSO
❮ 16–17 Addition and Subtraction
Division 22–25 ❯
Decimals 44–45 ❯

MULTIPLICATION INVOLVES ADDING A NUMBER TO ITSELF A NUMBER OF TIMES. THE RESULT OF MULTIPLYING NUMBERS IS CALLED THE PRODUCT.

What is multiplication?

The second number in a multiplication sum is the number to be added to itself and the first is the number of times to add it. Here the number of rows of people is added together a number of times determined by the number of people in each row. This multiplication sum gives the total number of people in the group.



multiplication sign

9 × 13

there are 9 rows of people

there are 13 people in each row

△ **How many people?**
The number of rows (9) is multiplied by the number of people in each row (13). The total number of people is 117.

this sum means 13 added to itself 9 times

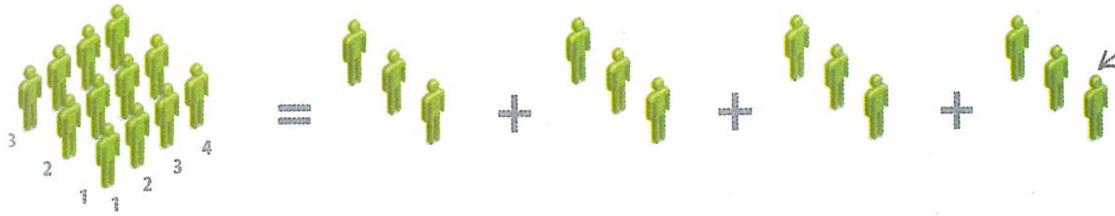
$9 \times 13 = 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 = \mathbf{117}$

product of 9 and 13 is 117

Works both ways

It does not matter which order numbers appear in a multiplication sum because the answer will be the same either way. Two methods of the same multiplication are shown here.

$$4 \times 3 = 3 + 3 + 3 + 3 = 12$$



$$3 \times 4 = 4 + 4 + 4 = 12$$

Multiplying by 10, 100, 1,000

Multiplying whole numbers by 10, 100, 1,000, and so on involves adding one zero (0), two zeroes (00), three zeroes (000), and so on to the right of the start number.

add 0 to end of start number

$$34 \times 10 = 340$$

add 00 to end of start number

$$72 \times 100 = 7{,}200$$

add 000 to end of start number

$$18 \times 1{,}000 = 18{,}000$$

Patterns of multiplication

There are quick ways to multiply two numbers, and these patterns of multiplication are easy to remember. The table shows patterns involved in multiplying numbers by 2, 5, 6, 9, 12, and 20.

PATTERNS OF MULTIPLICATION

To multiply	How to do it	Example to multiply
2	add the number to itself	$2 \times 11 = 11 + 11 = 22$
5	the last digit of the number follows the pattern 5, 0, 5, 0	5, 10, 15, 20
6	multiplying 6 by any even number gives an answer that ends in the same last digit as the even number	$6 \times 12 = 72$ $6 \times 8 = 48$
9	multiply the number by 10, then subtract the number	$9 \times 7 = 10 \times 7 - 7 = 63$
12	multiply the original number first by 10, then multiply the original number by 2, and then add the two answers	$12 \times 10 = 120$ $12 \times 2 = 24$ $120 + 24 = 144$
20	multiply the number by 10 then multiply the answer by 2	$14 \times 20 =$ $14 \times 10 = 140$ $140 \times 2 = 280$

MULTIPLES

When a number is multiplied by any whole number the result (product) is called a multiple. For example, the first six multiples of the number 2 are 2, 4, 6, 8, 10, and 12. This is because $2 \times 1 = 2$, $2 \times 2 = 4$, $2 \times 3 = 6$, $2 \times 4 = 8$, $2 \times 5 = 10$, and $2 \times 6 = 12$.

MULTIPLES OF 3

$$3 \times 1 = 3$$
$$3 \times 2 = 6$$
$$3 \times 3 = 9$$
$$3 \times 4 = 12$$
$$3 \times 5 = 15$$

first five multiples of 3

MULTIPLES OF 8

$$8 \times 1 = 8$$
$$8 \times 2 = 16$$
$$8 \times 3 = 24$$
$$8 \times 4 = 32$$
$$8 \times 5 = 40$$

first five multiples of 8

MULTIPLES OF 12

$$12 \times 1 = 12$$
$$12 \times 2 = 24$$
$$12 \times 3 = 36$$
$$12 \times 4 = 48$$
$$12 \times 5 = 60$$

first five multiples of 12

Common multiples

Two or more numbers can have multiples in common. Drawing a grid, such as the one on the right, can help find the common multiples of different numbers. The smallest of these common numbers is called the lowest common multiple.

24

Lowest common multiple
The lowest common multiple of 3 and 8 is 24 because it is the smallest number that both multiply into

multiples of 3

multiples of 8

multiples of 3 and 8

▷ **Finding common multiples**
Multiples of 3 and multiples of 8 are highlighted on this grid. Some multiples are common to both numbers.

Short multiplication

Multiplying a large number by a single-digit number is called short multiplication. The smaller number is placed below the larger one and aligned under the units column of the larger number.

$$\begin{array}{r} 196 \\ \times\quad 7 \\ \hline 2 \\ \hline 4 \end{array}$$

6 written in units column

2 written in units column

4 carried to tens column

To multiply 196 and 7, first multiply the units 7 and 6. The product is 42, the 4 of which is carried.

$$\begin{array}{r} 196 \\ \times\quad 7 \\ \hline 72 \\ \hline 64 \end{array}$$

9 written in tens column

7 written in tens column

6 carried to hundreds column

Next, multiply 7 and 9, the product of which is 63. The carried 4 is added to 63 to get 67.

$$\begin{array}{r} 196 \\ \times\quad 7 \\ \hline 1,372 \\ \hline 64 \end{array}$$

1 written in hundreds column

3 written in hundreds column; 1 written in thousands column

1,372 is final answer

Finally, multiply 7 and 1. Add the product (7) to the carried 6 to get 13, giving a final product of 1,372.

Long multiplication

Multiplying two numbers that both contain at least two digits is called long multiplication. The numbers are placed one above the other, in columns arranged according to their value (units, tens, hundreds, and so on).

$$\begin{array}{r} 428 \\ \times\ 111 \\ \hline 428 \end{array}$$

428 is multiplied by 1

First, multiply 428 by 1 in the units column. Work digit by digit from right to left so 8 × 1, 2 × 1, and then 4 × 1.

$$\begin{array}{r} 428 \\ \times\ 111 \\ \hline 428 \\ 4,280 \end{array}$$

428 is multiplied by 10

add 0 when multiplying by 10

Multiply 428 by 1 in the tens column, working digit by digit. Remember to add 0 to the product when multiplying by 10.

$$\begin{array}{r} 428 \\ \times\ 111 \\ \hline 428 \\ 4,280 \\ 42,800 \end{array}$$

428 is multiplied by 100

add 00 when multiplying by 100

Multiply 428 by 1 in the hundreds column, digit by digit. Add 00 to the product when multiplying by 100.

$$\begin{array}{r} 428 \\ \times\ 111 \\ \hline 428 \\ 4,280 \\ +\ 42,800 \\ \hline =\ 47,508 \end{array}$$

Add together the products of the three multiplications. The answer is 47,508.

LOOKING CLOSER

Box method of multiplication

The long multiplication of 428 and 111 can be broken down into simple multiplications with the help of a table or a box. Each number is reduced to its hundreds, tens, and units, and multiplied by the other.

▷ **The final step**
Add together the nine multiplications to find the final answer.

Not taught to students

111 WRITTEN IN 100S, 10S, AND UNITS / 428 WRITTEN IN 100S, 10S, AND UNITS	400	20	8
100	400 × 100 = 40,000	20 × 100 = 2,000	8 × 100 = 800
10	400 × 10 = 4,000	20 × 10 = 200	8 × 10 = 80
1	400 × 1 = 400	20 × 1 = 20	8 × 1 = 8

$$\begin{array}{r} 40,000 \\ 2,000 \\ 800 \\ 4,000 \\ 200 \\ 80 \\ 400 \\ 20 \\ +\quad 8 \\ \hline =\ 47,508 \end{array}$$

this is the final answer

Division

DIVISION INVOLVES FINDING OUT HOW MANY TIMES ONE NUMBER GOES INTO ANOTHER NUMBER.

SEE ALSO
❮ 16–17 Addition and subtraction
❮ 18–21 Multiplication
Ratio and proportion 56–59 ❯

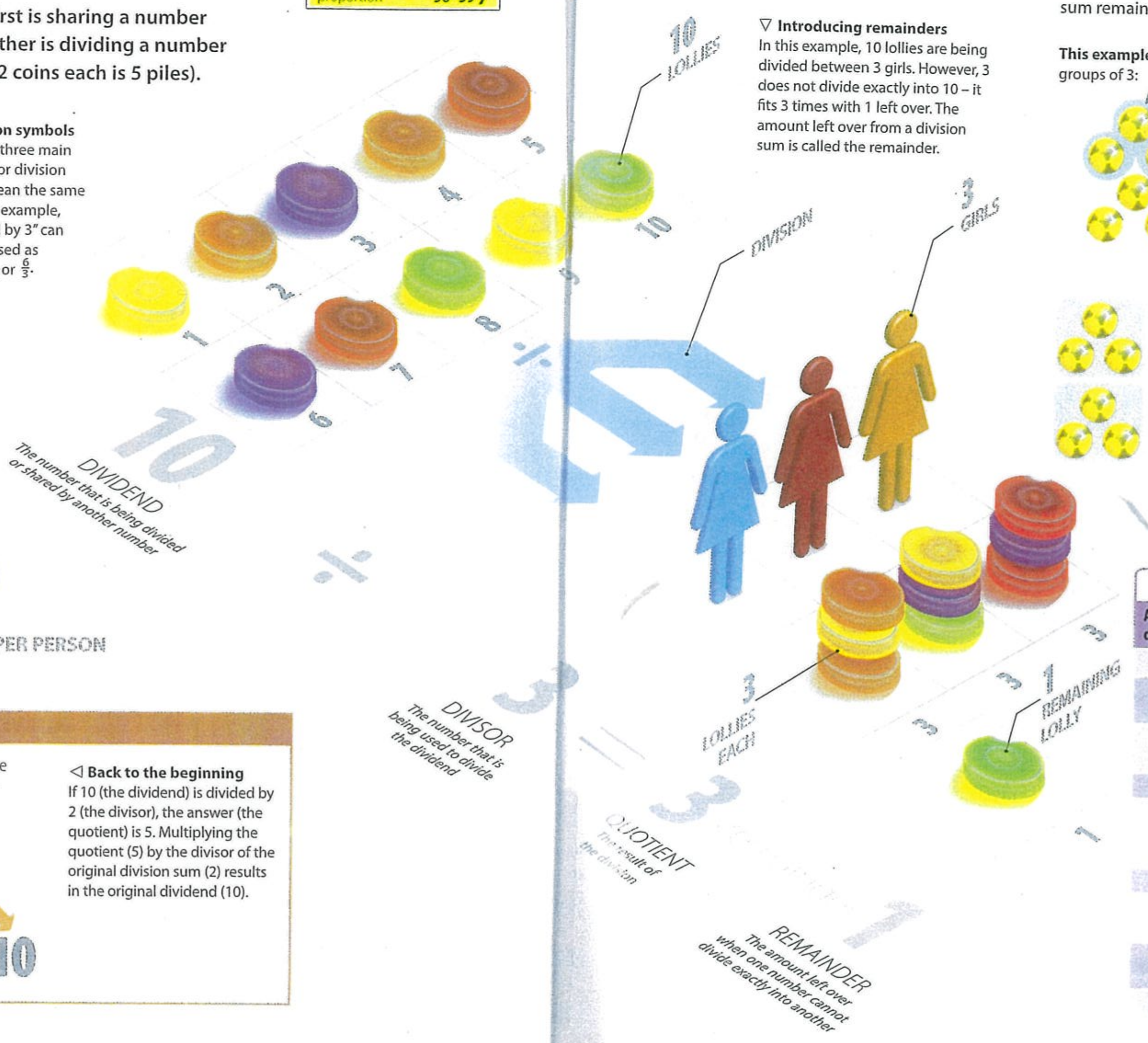
There are two ways to think about division. The first is sharing a number out equally (10 coins to 2 people is 5 each). The other is dividing a number into equal groups (10 coins into piles containing 2 coins each is 5 piles).

How division works

Dividing one number by another finds out how many times the second number (the divisor) fits into the first (the dividend). For example, dividing 10 by 2 finds out how many times 2 fits into 10. The result of the division is known as the quotient.

◁ **Division symbols**
There are three main symbols for division that all mean the same thing. For example, "6 divided by 3" can be expressed as $6 \div 3$, 6/3, or $\frac{6}{3}$.

▽ **Division as sharing**
Sharing equally is one type of division. Dividing four sweets equally between two people means that each person gets the same number of sweets: two each.



▽ **Introducing remainders**
In this example, 10 lollies are being divided between 3 girls. However, 3 does not divide exactly into 10 – it fits 3 times with 1 left over. The amount left over from a division sum is called the remainder.

LOOKING CLOSER

How division is linked to multiplication

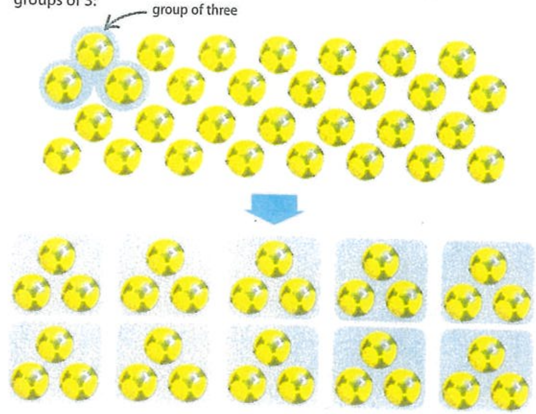
Division is the direct opposite or "inverse" of multiplication, and the two are always connected. If you know the answer to a particular division, you can form a multiplication from it and vice versa.

◁ **Back to the beginning**
If 10 (the dividend) is divided by 2 (the divisor), the answer (the quotient) is 5. Multiplying the quotient (5) by the divisor of the original division sum (2) results in the original dividend (10).

Another approach to division

Instead of thinking of it as sharing out a number, division can also be viewed as finding out how many groups of the second number (divisor) are contained in the first number (dividend). The division sum remains the same in both sharing and grouping.

This example shows 30 footballs, which are to be divided into groups of 3:



There are exactly 10 groups of 3 footballs, with no remainder, so $30 \div 3 = 10$.

DIVISION TIPS

A number is divisible by	If...	Examples
2	the last digit is an even number	12, 134, 5,000
3	the sum of all digits when added together is divisible by 3	18 1+8 = 9
4	the number formed by the last two digits is divisible by 4	732 32 ÷ 4 = 8
5	the last digit is 5 or 0	25, 90, 835
6	the last digit is even and the sum of its digits when added together is divisible by 3	3,426 3+4+2+6 = 15
7	no simple divisibility test	
8	the number formed by the last three digits is divisible by 8	7,536 536 ÷ 8 = 67
9	the sum of all of its digits is divisible by 9	6,831 6+8+3+1 = 18
10	the number ends in 0	30, 150, 4,270

Short division

Short division is used to divide one number (the dividend) by another whole number (the divisor) that is less than 10.

start on the left with the first 3 (divisor)

dividing line

$$\begin{array}{r} 1 \\ 3\overline{)396} \end{array}$$

396 is the dividend

Divide the first 3 into 3. It fits once exactly, so put a 1 above the dividing line, directly above the 3 of the dividend.

$$\begin{array}{r} 13 \\ 3\overline{)396} \end{array}$$

Move to the next column and divide 3 into 9. It fits three times exactly, so put a 3 directly above the 9 of the dividend.

result is 132

$$\begin{array}{r} 132 \\ 3\overline{)396} \end{array}$$

Divide 3 into 6, the last digit of the dividend. It goes twice exactly, so put a 2 directly above the 6 of the dividend.

Carrying numbers

When the result of a division gives a whole number and a remainder, the remainder can be carried over to the next digit of the dividend.

start on the left

divisor

$$5\overline{)2{,}765}$$

2,765 is the dividend

Start with number 5. It does not divide into 2 as it is a larger number. Instead, 5 will need to be divided into the first two digits of the dividend.

divide 5 into first 2 digits of dividend

carry remainder 2 to next digit of dividend

$$\begin{array}{r} 5 \\ 5\overline{)2{,}7{}^{2}65} \end{array}$$

Divide 5 into 27. The result is 5 with a remainder of 2. Put 5 directly above the 7 and carry the remainder.

carry remainder 1 to next digit of dividend

$$\begin{array}{r} 55 \\ 5\overline{)2{,}7{}^{2}6{}^{1}5} \end{array}$$

Divide 5 into 26. The result is 5 with a remainder of 1. Put 5 directly above the 6 and carry the remainder 1 to the next digit of the dividend.

the result is 553

$$\begin{array}{r} 553 \\ 5\overline{)2{,}7{}^{2}6{}^{1}5} \end{array}$$

Divide 5 into 15. It fits three times exactly, so put 3 above the dividing line, directly above the final 5 of the dividend.

LOOKING CLOSER

Converting remainders

When one number will not divide exactly into another, the answer has a remainder. Remainders can be converted into decimals, as shown below.

remainder

$$\begin{array}{r} 22\ \text{r}\ 2 \\ 4\overline{)9{}^{1}0} \end{array}$$

$$\begin{array}{r} 22. \\ 4\overline{)9{}^{1}0.0} \end{array}$$

Remove the remainder, 2 in this case, leaving 22. Add a decimal point above and below the dividing line. Next, add a zero to the dividend after the decimal point.

$$\begin{array}{r} 22. \\ 4\overline{)9{}^{1}0.{}^{2}0} \end{array}$$

Carry the remainder (2) from above the dividing line to below the line and put it in front of the new zero.

$$\begin{array}{r} 22.5 \\ 4\overline{)9{}^{1}0.{}^{2}0} \end{array}$$

Divide 4 into 20. It goes 5 times exactly, so put a 5 directly above the zero of the dividend and after the decimal point.

LOOKING CLOSER

Making division simpler

To make a division easier, sometimes the divisor can be split into factors. This means that a number of simpler divisions can be done.

$816 \div 6$ divisor is 6, which is 2×3. Splitting 6 into 2 and 3 simplifies the sum

result is 136

$816 \div 2 = 408$ ➡ $408 \div 3 = 136$

divide by first factor of divisor

divide by second factor of divisor

This method of splitting the divisor into factors can also be used for more difficult divisions.

$405 \div 15$ splitting 15 into 5 and 3, which multiply to make 15, simplifies the sum

result is 27

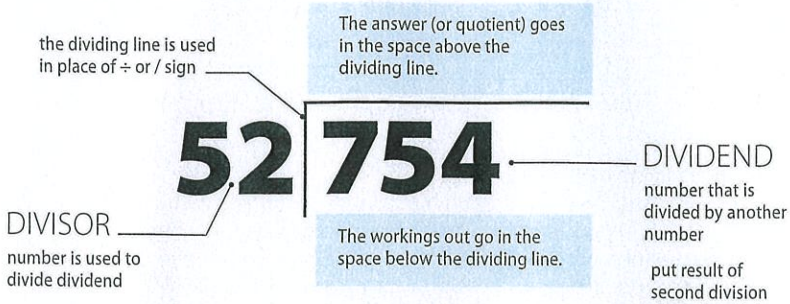
$405 \div 5 = 81$ ➡ $81 \div 3 = 27$

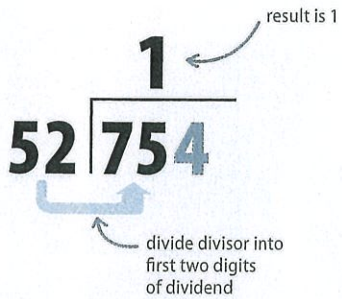
divide by first factor of divisor

divide result by second factor of divisor

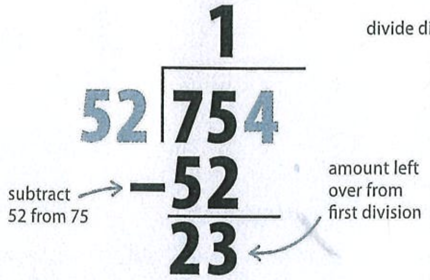
Long division

Long division is usually used when the divisor is at least two digits long and the dividend is at least 3 digits long. Unlike short division, all the workings out are written out in full below the dividing line. Multiplication is used for finding remainders. A long division sum is presented in the example on the right.

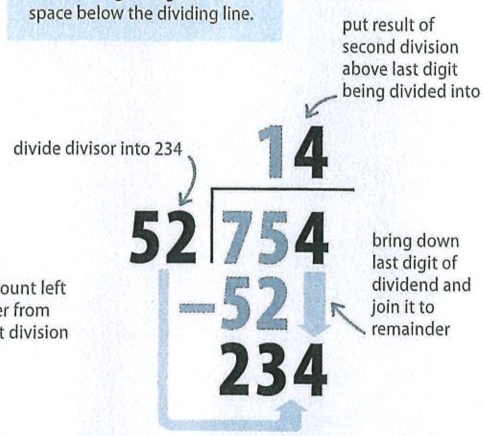




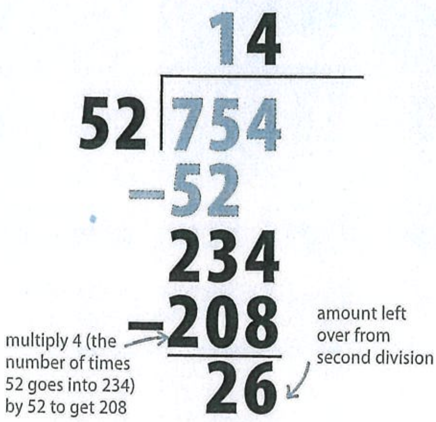
Begin by dividing the divisor into the first two digits of the dividend. 52 fits into 75 once, so put a 1 above the dividing line, aligning it with the last digit of the number being divided.



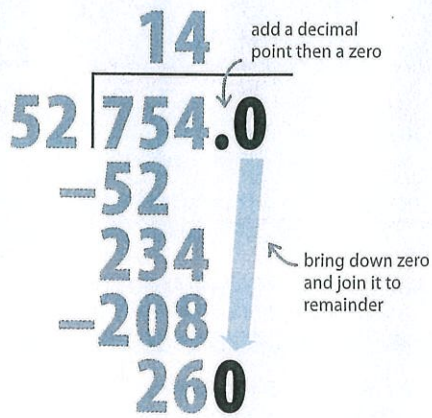
Work out the first remainder. The divisor 52 does not divide into 75 exactly. To work out the amount left over (the remainder), subtract 52 from 75. The result is 23.



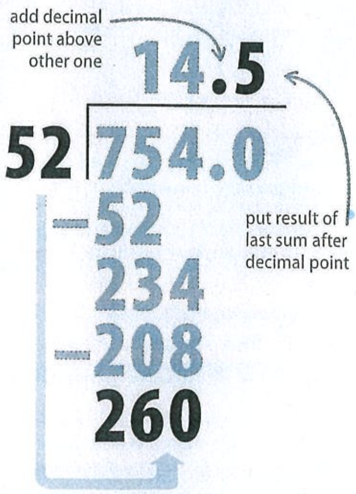
Now, bring down the last digit of the dividend and place it next to the remainder to form 234. Next, divide 234 by 52. It goes four times, so put a 4 next to the 1.



Work out the second remainder. The divisor, 52, does not divide into 234 exactly. To find the remainder, multiply 4 by 52 to make 208. Subtract 208 from 234, leaving 26.



There are no more whole numbers to bring down, so add a decimal point after the dividend and a zero after it. Bring down the zero and join it to the remainder 26 to form 260.



Put a decimal point after the 14. Next, divide 260 by 52, which goes five times exactly. Put a 5 above the dividing line, aligned with the new zero in the dividend.

Units of measurement

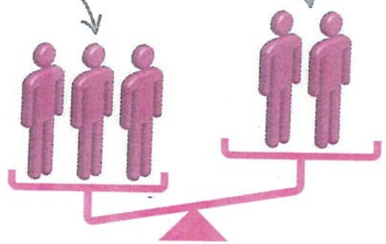
UNITS OF MEASUREMENT ARE STANDARD SIZES USED TO MEASURE TIME, MASS, AND LENGTH.

SEE ALSO
Volumes 154–155 ›
Formulas 177–179 ›
Reference 242–245 ›

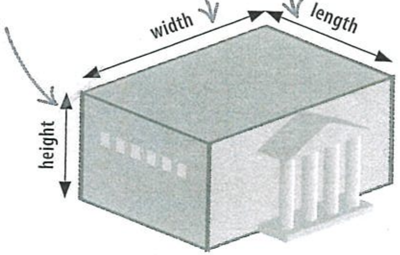
Basic units

A unit is any agreed or standardized measurement of size. This allows quantities to be accurately measured. There are three basic units: time, weight (including mass), and length.

△ **Time**
Time is measured in milliseconds, seconds, minutes, hours, days, weeks, months, and years. Different countries and cultures may have calendars which start a new year at a different time.



△ **Weight and mass**
Weight is how heavy something is in relation to the force of gravity acting upon it. Mass is the amount of matter that makes up the object. Both are measured in the same units, such as grams and kilograms, or ounces and pounds.

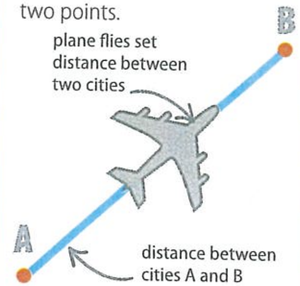


△ **Length**
Length is how long something is. It is measured in centimetres, metres, and kilometres in the metric system, or in inches, feet, yards, and miles in the imperial system (see pp.242–245).

LOOKING CLOSER

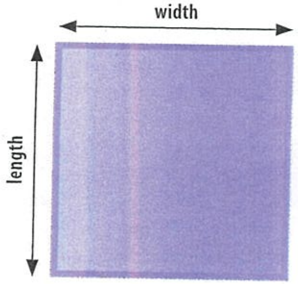
Distance

The distance is the amount of space between two points. It expresses length, but is also used to describe a journey, which is not always the most direct route between two points.



Compound measures

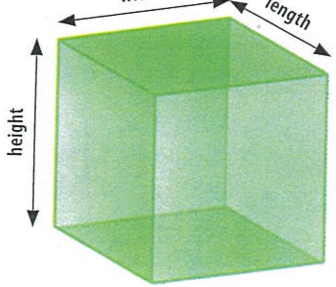
A compound unit is made up of more than one of the basic units, including using the same unit repeatedly. Examples include area, volume, speed, and density.



◁ **Area**
Area is measured in squared units. The area of a square is the product of its length and width; if they were both measured in metres (m), its area would be m × m, which is written as m^2.

area = length × width

area is made up of two of the same units, as width is also a length



◁ **Volume**
Volume is measured in cubed units. The volume of a cuboid is the product of its height, width, and length; if they were all measured in metres (m), its area would be m × m × m, or m^3.

volume = length × width × height

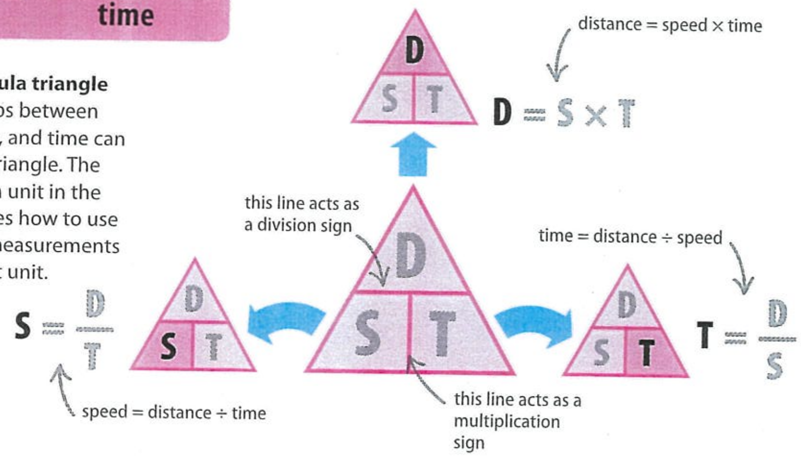
volume is a compound of three of the same units, as width and height are technically lengths

Speed

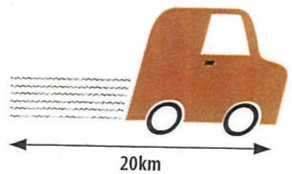
Speed measures the distance (length) travelled in a given time. This means that the formula for measuring speed is length ÷ time. If this is measured in kilometres and hours, the unit for speed will be km/h.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

▷ **Speed formula triangle**
The relationships between speed, distance, and time can be shown in a triangle. The position of each unit in the triangle indicates how to use the other two measurements to calculate that unit.



▷ **Finding speed**
A van travels 20km in 20 minutes. From this information its speed in km/h can be found.



divide 20 by 60 to find its value in hours

$$20 \text{ minutes} = \frac{20}{60} = \frac{1}{3} \text{ hour}$$

First, convert the minutes into hours. To convert minutes into hours, divide them by 60, then cancel the fraction – divide the top and bottom numbers by 20. This gives an answer of $^1/_3$ hour.

distance is 20km

$$S = \frac{D}{T} = 60\text{km/h}$$

time is $^1/_3$ hour

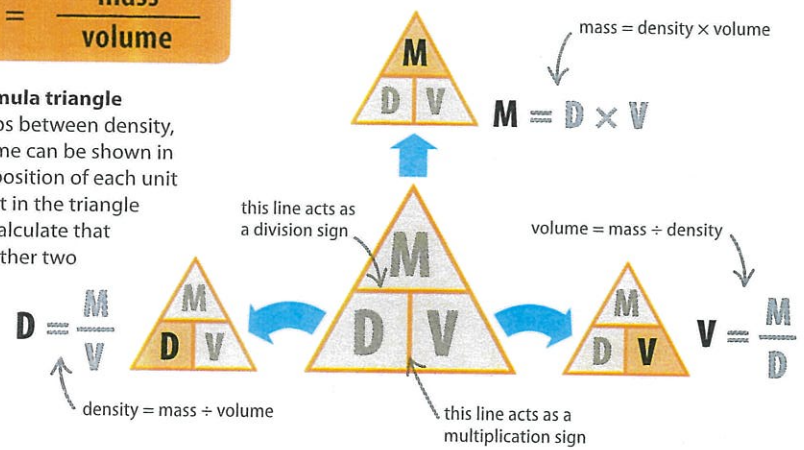
Then, substitute the values for distance and time into the formula for speed. Divide the distance (20km) by the time ($^1/_3$ hour) to find the speed, in this case 60 km/h.

Density

Density measures how much matter is packed into a given volume of a substance. It involves two units – mass and volume. The formula for measuring density is mass ÷ volume. If this is measured in grams and centimetres, the unit for density will be g/cm^3.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

▷ **Density formula triangle**
The relationships between density, mass, and volume can be shown in a triangle. The position of each unit of measurement in the triangle shows how to calculate that unit using the other two measurements.



density of lead is constant, regardless of mass

▷ **Finding volume**
Lead has a density of 0.0113kg/cm^3. With this measurement, the volume of a lead weight that has a mass of 0.5kg can be found.

mass is 0.5kg

$$V = \frac{M}{D} = 44.25\text{cm}^3$$

density is 0.0113kg/cm^3

△ **Using the formula**
Substitute the values for mass and density into the formula for volume. Divide the mass (0.5kg) by the density (0.0113kg/cm^3) to find the volume, in this case 44.25cm^3.

Mental maths

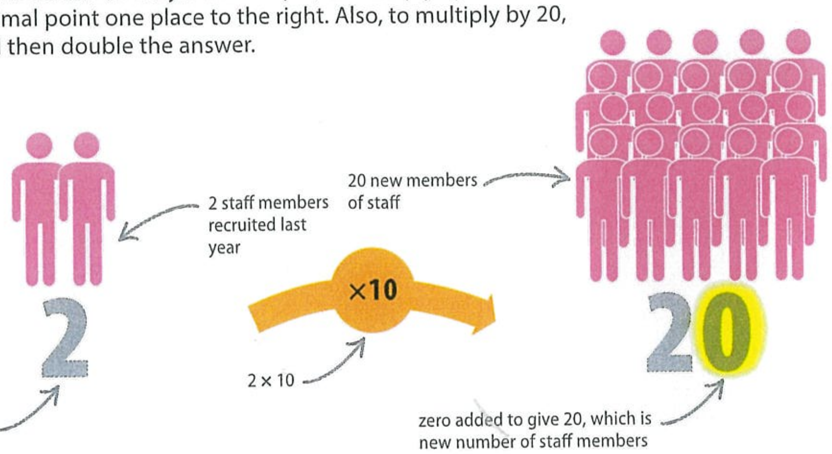
EVERYDAY PROBLEMS CAN BE SIMPLIFIED SO THAT THEY CAN BE EASILY DONE WITHOUT USING A CALCULATOR.

SEE ALSO
❮ 18–21 Multiplication
❮ 22–25 Division
Using a calculator 72–73 ❯

MULTIPLICATION

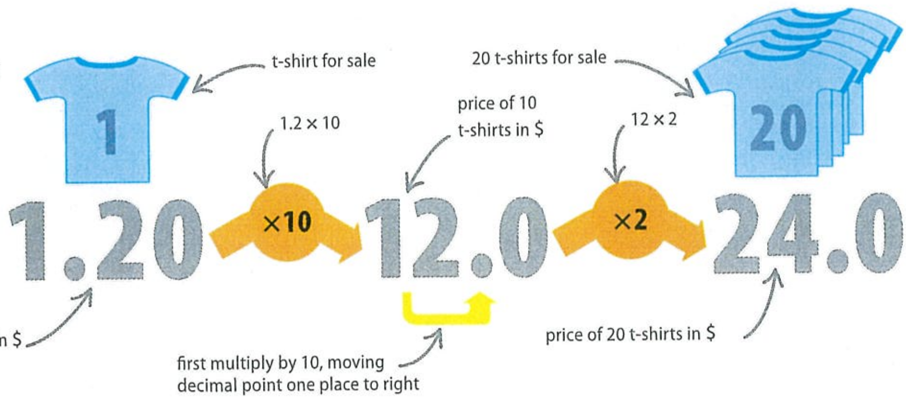
Multiplying by some numbers can be easy. For example, to multiply by 10 either add a 0 or move the decimal point one place to the right. Also, to multiply by 20, again multiply by 10 and then double the answer.

▷ **Multiplying by 10**
A sports club hired 2 people last year, but this year it needs to hire 10 times that amount. How many staff members will it recruit this year?



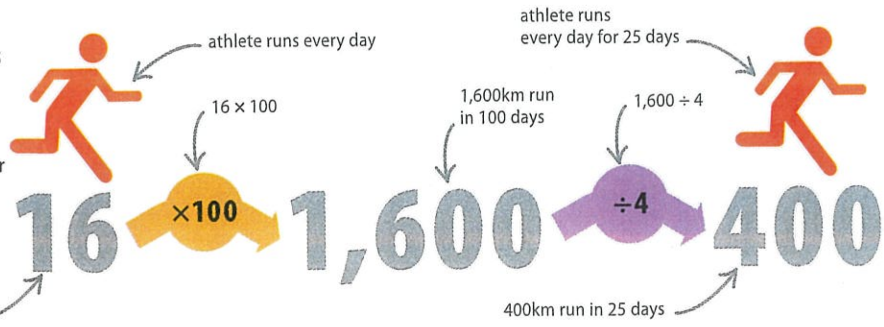
◁ **Finding the answer**
To multiply 2 by 10 add a 0 to the 2. Multiplying 2 people by 10 results in an answer of 20.

▷ **Multiplying by 20**
A shop is selling t-shirts for the price of $1.20 each. How much will the price be for 20 t-shirts?



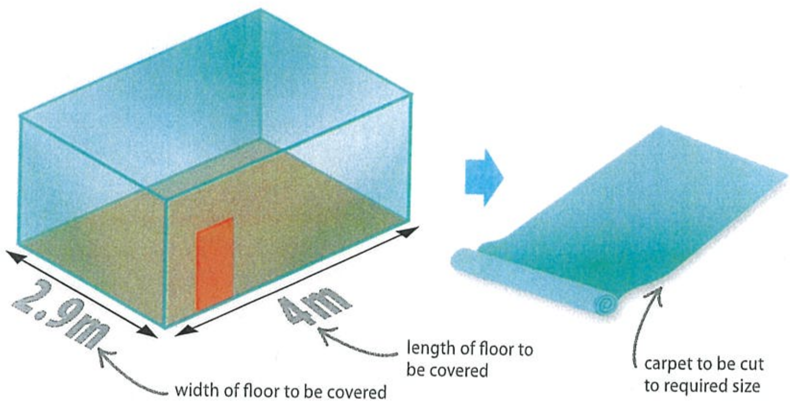
◁ **Finding the answer**
First multiply the price by 10, here by moving the decimal point one place to the right, and then double that to give the final price of $24.

▷ **Multiplying by 25**
An athlete runs 16km a day. If the athlete runs the same distance every day for 25 days, how far will he run in total?



◁ **Finding the answer**
First multiply the 16km for one day by 100, to give 1,600km for 100 days, then divide by 4 to give the answer over 25 days.

▽ Multiplication using decimals
Decimals appear to complicate the problem, but they can be ignored until the final stage. Here the amount of carpet required to cover a floor needs to be calculated.



LOOKING CLOSER

Checking the answer

As 2.9 is almost 3, multiplying 3 × 4 is a good way to check that the calculation to 2.9 × 4 is correct.

symbol for approximately equal to

$$2.9 \approx 3 \text{ and } 3 \times 4 = 12$$

close to real answer of 11.6

$$\text{so } 2.9 \times 4 \approx 12$$

width of floor, length

$2.9 \times 4 \rightarrow 29 \times 4$

width with decimal point removed; length

30 easier to work with than 29

30×4

1×4

1 × 4 to be subtracted from 30 × 4

120 ← sum of 30 × 4

$-\ 4$ ← sum of 1 × 4

result of 120 – 4 → 116

decimal point moved one place to left to give answer 11.6

$116 \rightarrow 11.6$

First, take away the decimal point from the 2.9 to make the calculation 29 × 4.

Change 29 × 4 to 30 × 4, as it is easier to work out. Write 1 × 4 below as it is the difference between 29 × 4 and 30 × 4.

Subtract 4 (product of 1 × 4) from 120 (product of 30 × 4) to give the answer of 116 (product of 29 × 4).

Move the decimal point one place to the left as it was moved one place to the right in the first step.

Top tricks

The multiplication tables of several numbers reveal patterns of multiplications. Here are two good mental tricks to remember when multiplying the 9 and 11 times tables.

multipliers 1 to 10

9 TIMES TABLE

1	2	3	4	5	6	7	8	9	10
9	18	27	36	45	54	63	72	81	90

1 + 8 = 9 — 7 + 2 = 9 — multiples of 9

△ Two digits are added together
The two digits that make up the first 10 multiples of 9 each add up to 9. The first digit of the multiple (such as 1, in 18) is always 1 less than the multiplier (2).

multipliers 1 to 9

11 TIMES TABLE

1	2	3	4	5	6	7	8	9
11	22	33	44	55	66	77	88	99

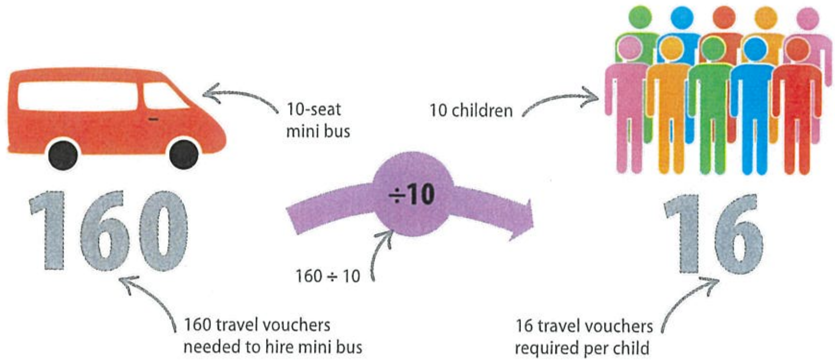
11 × 3 = 33, or 3 written twice — 11 × 7 = 77, or 7 written twice — multiples of 11

△ Digit is written twice
To multiply by 11, merely repeat the two multipliers together. For example, 4 × 11 is two 4s or 44. It works all the way up to 9 × 11 = 99, which is 9 written twice.

DIVISION

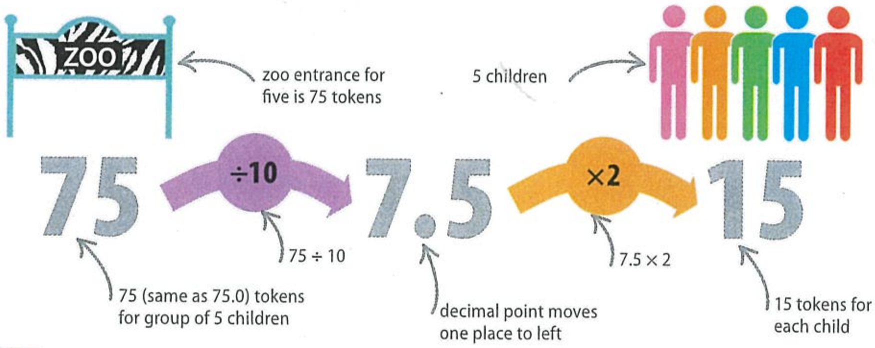
Dividing by 10 or 5 is straightforward. To divide by 10, either delete a 0 or move the decimal point one place to the left. To divide by 5, again divide by 10 and then double the answer. Using these rules, work out the divisions in the following two examples.

▷ **Dividing by 10**
In this example, 160 travel vouchers are needed to hire a 10-seat mini bus. How many travel vouchers are needed for each of the 10 children to travel on the bus?



◁ **How many each?**
To find the number of travel vouchers for each child, divide the total of 160 by 10 by deleting a 0 from the 160. It gives the answer of 16 travel vouchers each.

▷ **Dividing by 5**
The cost of admission to a zoo for a group of five children is 75 tokens. How many tokens are needed for 1 of the 5 children to enter the zoo?



◁ **How many each?**
To find the admission for 1 child, divide the total of 75 by 10 (by moving a decimal point in 75 one place to the left) to give 7.5, and then double that for the answer of 15.

LOOKING CLOSER

Top tips

There are various mental tricks to help with dividing larger or more complicated numbers. In the three examples below, there are tips on how to check whether very large numbers can be divided by 3, 4, and 9.

▷ **Divisible by 3**
Add together all of the digits in the number. If the total is divisible by 3, the original number is too.

original number: 1665233198172 ➡ $1+6+6+5+2+3+3+1+9+8+1+7+2=54$

digits add up to 54

$54 \div 3 = 18$, so the original number is divisible by 3

▷ **Divisible by 4**
If the last two digits are taken as one single number, and it is divisible by 4, the original number is too.

original number: 12345612345612345**6** ➡ $56 \div 4 = 14$

5 and 6 seen as one number: 56

$56 \div 4 = 14$, so the original number is divisible by 4

▷ **Divisible by 9**
Add together all of the digits in the number. If the total is divisible by 9, the original number is too.

original number: 1643951142 ➡ $1+6+4+3+9+5+1+1+4+2=36$

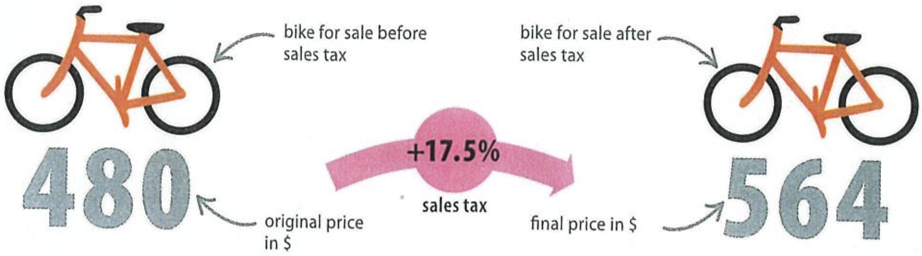
add together all digits of number, their sum is 36

$36 \div 9 = 4$, so the original number is divisible by 9

PERCENTAGES

A useful method of simplifying calculations involving percentages is to reduce one difficult percentage into smaller and easier-to-calculate parts. In the example below, the smaller percentages include 10% and 5%, which are easy to work out.

▷ **Adding 17.5 per cent**
Here a shop wants to charge $480 for a new bike. However, the owner of the shop has to add a sales tax of 17.5 per cent to the price. How much will it then cost?



sales tax

17.5% of 480

original price of bike

10% of 480 = 48
5% of 480 = 24
2.5% of 480 = 12

2.5% of 480 is half of 5% of 480, which is half of 10% of 480

	48
	24
+	12
	84

results added together

84 is 17.5% of 480

First, write down the percentage price increase required and the original price of the bike.

Next, reduce 17.5% into the easier stages of 10%, 5%, and 2.5% of $480, and calculate their values.

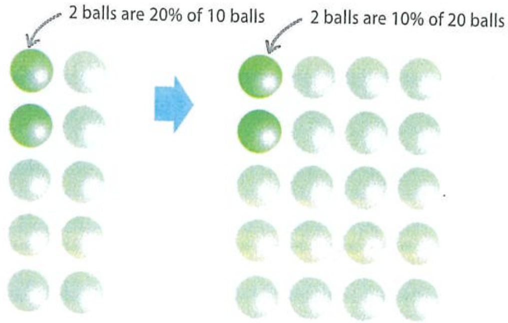
The sum of 48, 24, and 12 is 84, so $84 is added to $480 for a price of $564.

Switching

A percentage and an amount can both be "switched", to produce the same result with each switch. For example, 50% of 10, which is 5, is exactly the same as 10% of 50, which is 5 again.

20% is 2 of 10 balls — amount of balls is 10 — 10% is 2 of 20 balls — amount of balls is 20

20% of 10 = 10% of 20

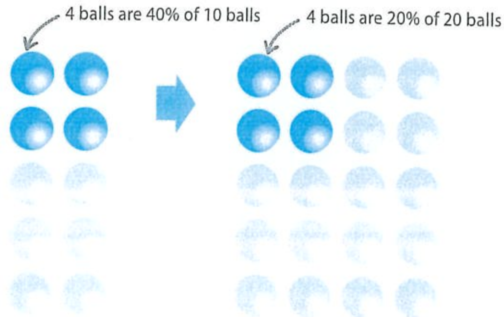


Progression

A progression involves dividing the percentage by a number and then multiplying the amount by the same number. For example, 40% of 10 is 4. Dividing this 40% by 2 and multiplying 10 by 2 results in 20% of 20, which is also 4.

40% is 4 of 10 balls — amount of balls is 10 — 20% is 4 of 20 balls — amount of balls is 20

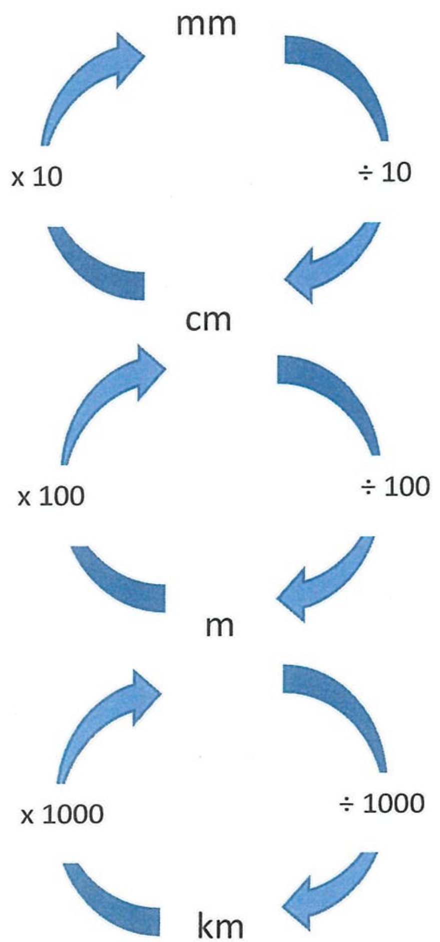
40% of 10 = 20% of 20

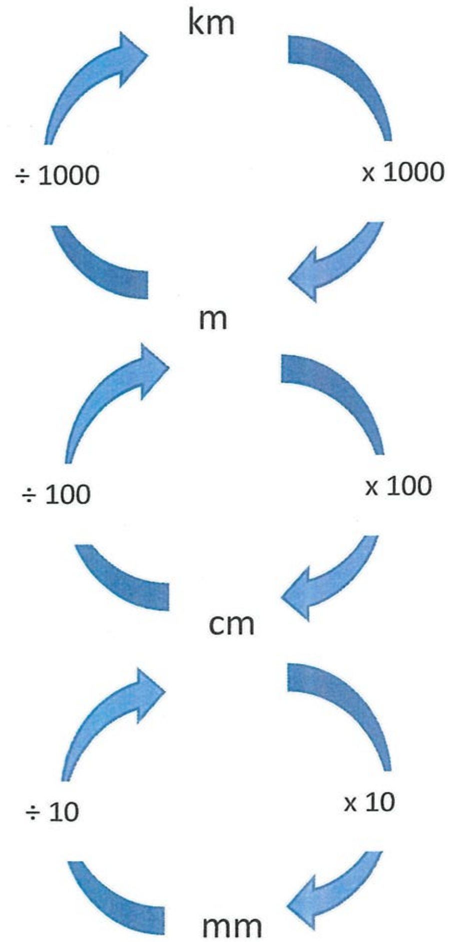


Measurement

Conversion

mm, cm and metres

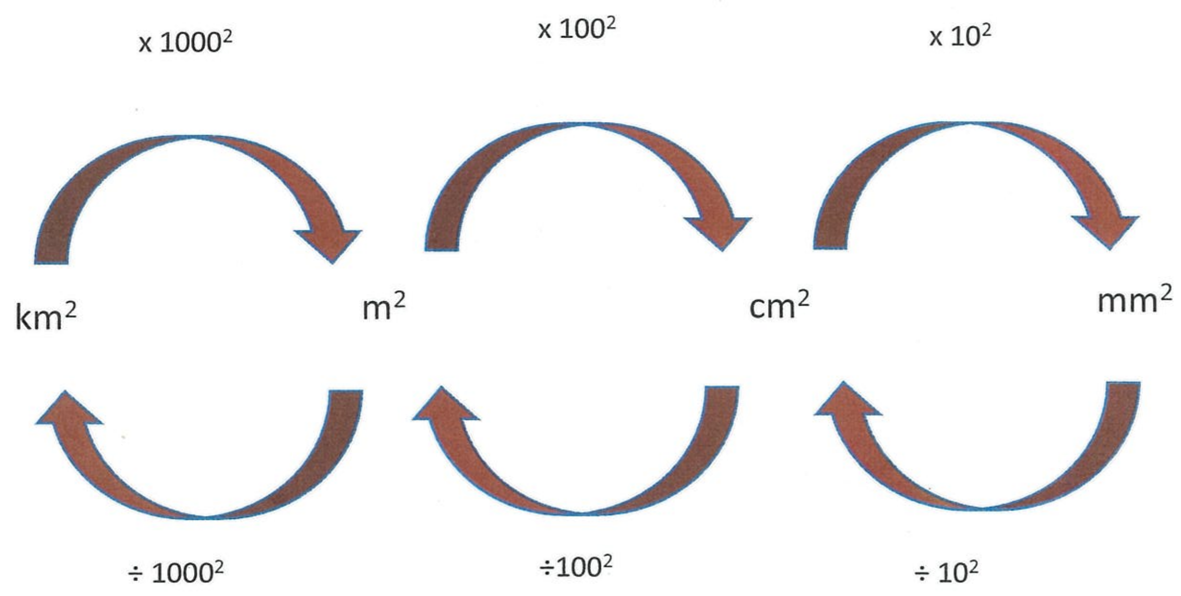




Measurement

Conversion

Area - km^2, m^2, cm^2, mm^2



Level 8

Chooses appropriate units of measurement for area and volume and converts from one unit to another. Recognises that the conversion factors for area of units are the squares of those for the corresponding linear units and for volume, units are the cubes of those for the corresponding linear units

Measurement

Perimeter

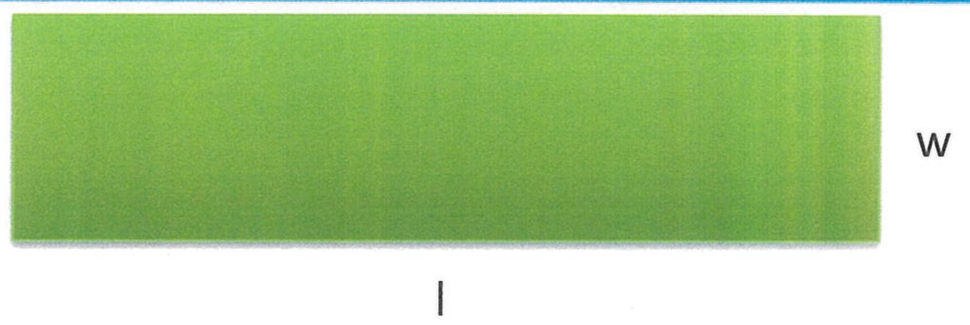
The word perimeter means 'a path that surrounds an area'. It comes from the Greek words *peri* meaning around and *metre* which means measure. Its first recorded usage was during the 15th century.

Perimeter is defined as the distance around a closed two-dimensional shape.

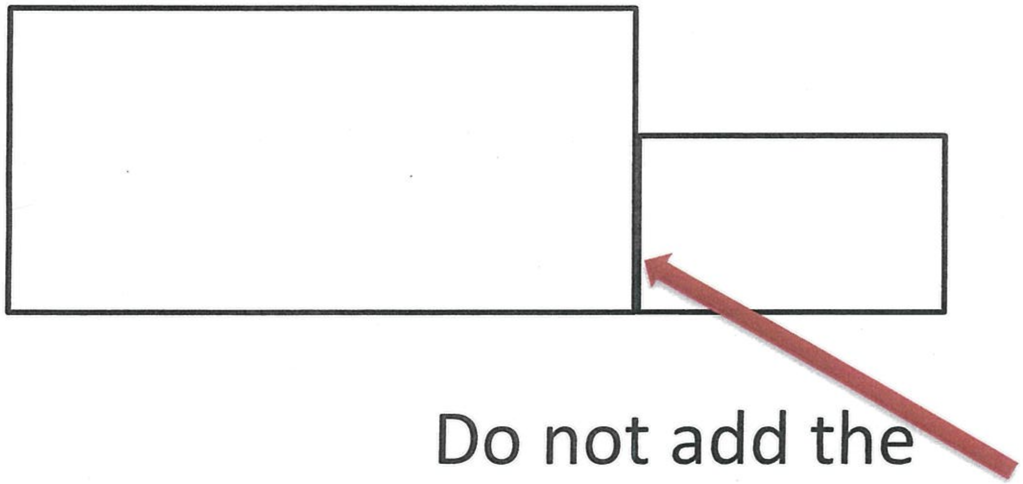
Level 5
Calculates the perimeter and area of rectangles using familiar metric units.
Explores efficient ways of calculating perimeters by adding the length and width together and doubling the result

Measurement

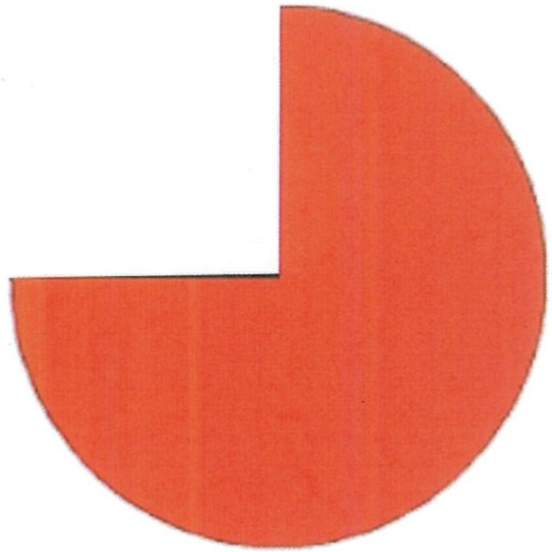
Perimeter



$$\text{Perimeter} = l + l + w + w$$
$$= 2l + 2w$$
$$= 2\,(l + w)$$



Do not add the internal line



12-04 Perimeter of composite shapes

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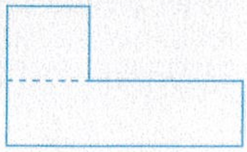
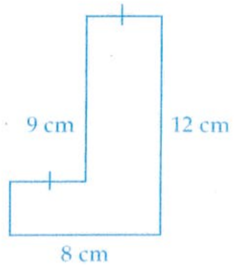
composite shape A shape made up of 2 or more shapes.
For example, a square and a rectangle.



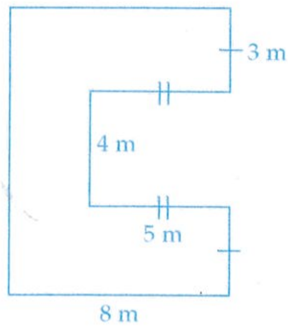
EXAMPLE 6

Find the perimeter of these shapes.

a



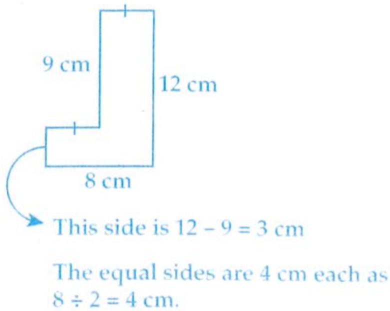
b



SOLUTION

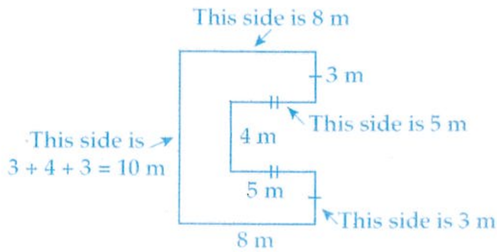
First, find the unknown sides.

a



$P = 3 + 4 + 9 + 4 + 12 + 8$
$= 40$ cm

b

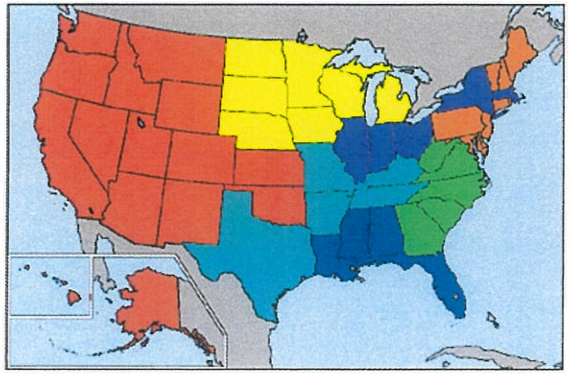


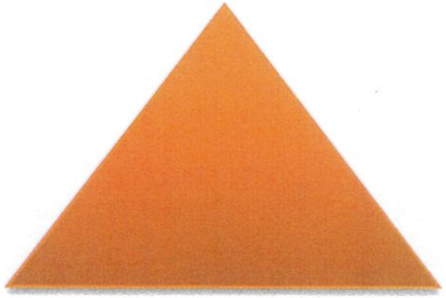
$P = 3 + 5 + 4 + 5 + 3 + 8 + 10 + 8$
$= 46$ m

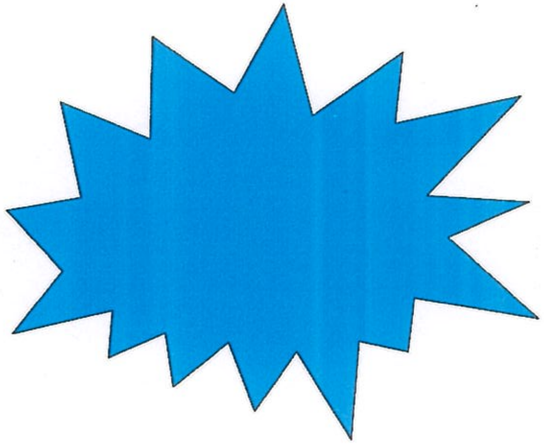
Measurement

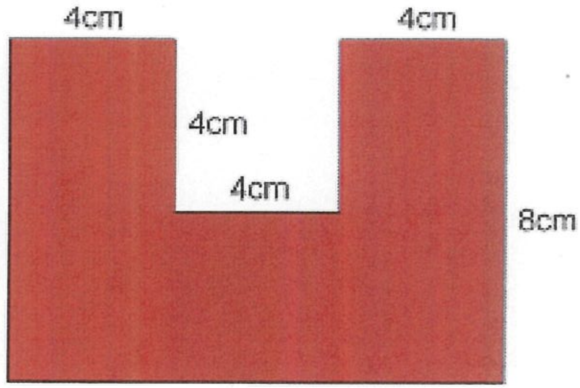
Area

Area is defined as a 2D space inside a region







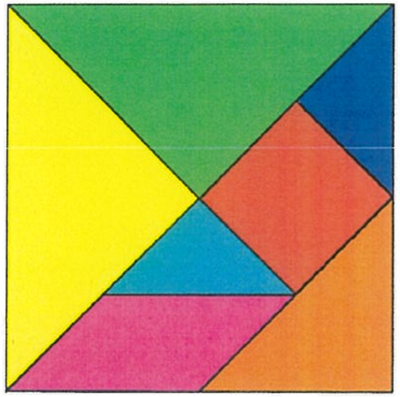


- Measured in units squared

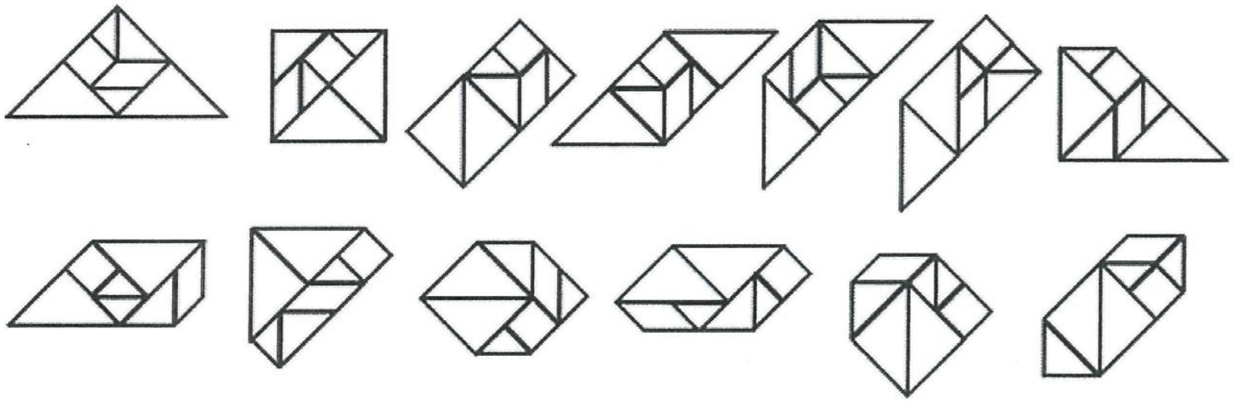
Measurement

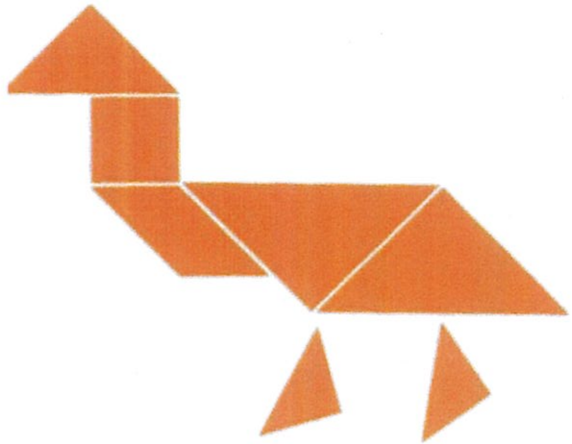
Area

Cutting a shape into different parts and reassembling it shows that different shapes can have the same area



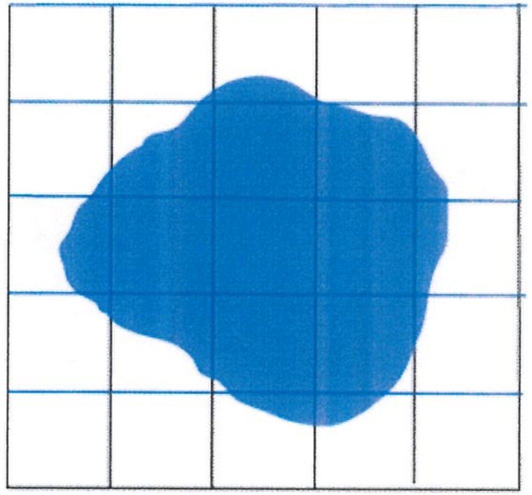
Use of tangrams



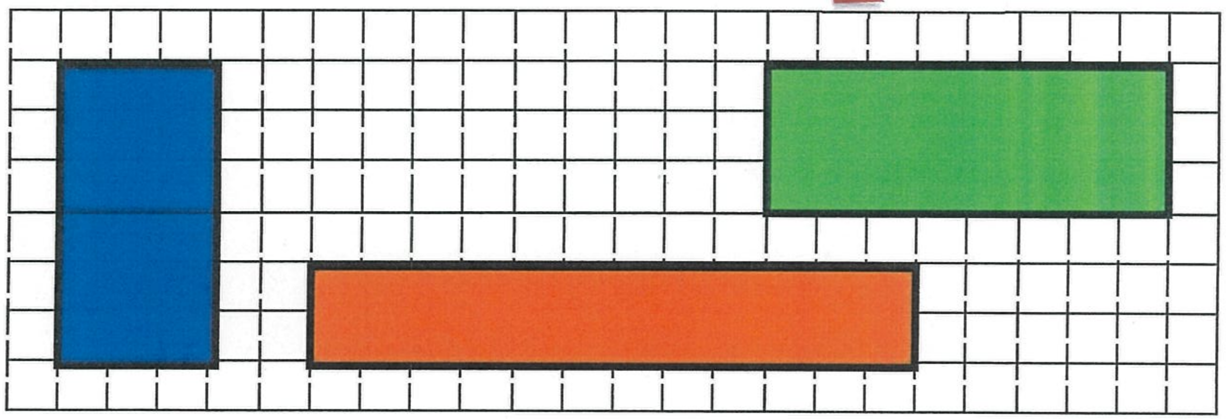


Measurement

Area



Level 2
Compares and orders several shapes and objects based on length, area, volume and capacity using appropriate uniform informal units

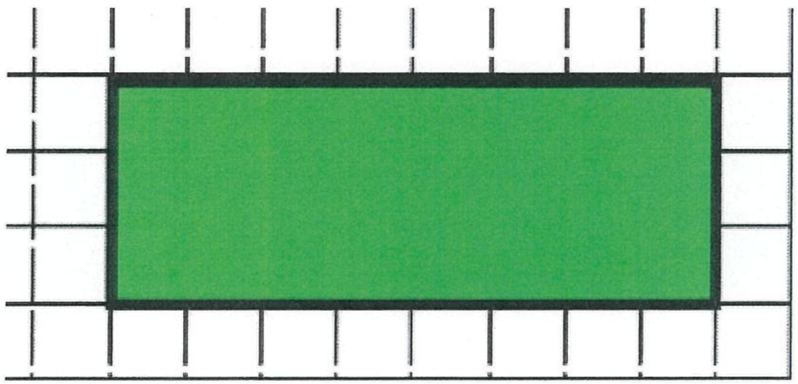


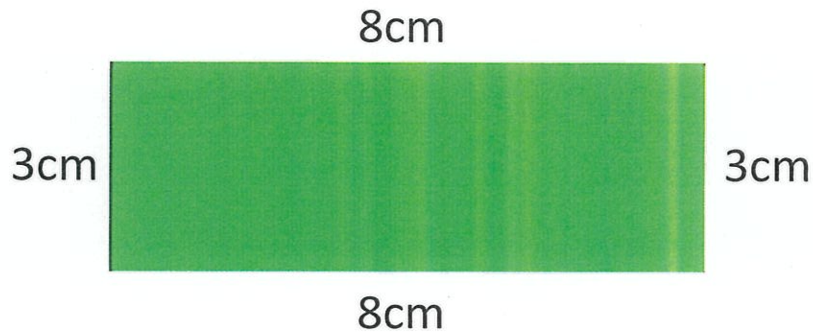
Level 4
Compares objects using familiar metric units of area (grid paper)

Measurement

Area

A 8 cm by 3 cm rectangle contains 8 × 3 = 24 squares, each with an area of 1 square centimetre. So the area of the rectangle is 24 square centimetres, or 24 cm^2





$$\begin{aligned} \text{Area} &= l \times w \\ &= 8\text{cm} \times 3\text{cm} \\ &= 24\ \text{cm}^2 \end{aligned}$$

10-01 Metric units for area

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area The amount of surface space inside a flat shape, measured in square units such as mm^2, cm^2, m^2 or km^2.

A **square millimetre** (mm^2) is the area of a square of length 1 mm, about the size of a grain of raw sugar or rock salt.

A **square centimetre** (cm^2) is the area of a square of length 1 cm, about the size of a face of a die.

1 cm = 10 mm

$1\ cm^2 = 10\ mm \times 10\ mm = 100\ mm^2$

A **square metre** (m^2) is the area of a square of length 1 m, about the size of the base of a shower floor.

1 m = 100 cm = 1000 mm

$1\ m^2 = 100\ cm \times 100\ cm = 10\ 000\ cm^2$

$1\ m^2 = 1000\ mm \times 1000\ mm = 1\ 000\ 000\ mm^2$

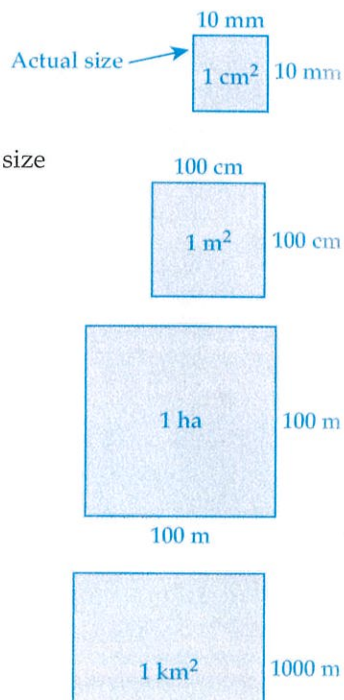
A **hectare** (ha) is the area of a square of length 100 m, about the size of two football fields.

$1\ ha = 100\ m \times 100\ m = 10\ 000\ m^2$

A **square kilometre** (km^2) is the area of a square of length 1 km, about the size of a theme park.

1 km = 1000 m

$1\ km^2 = 1000\ m \times 1000\ m = 1\ 000\ 000\ m^2$



$1\ cm^2 = 100\ mm^2$

$1\ m^2 = 10\ 000\ cm^2 = 1\ 000\ 000\ mm^2$

$1\ ha = 10\ 000\ m^2$

$1\ km^2 = 1\ 000\ 000\ m^2$

13-01 Area

WORDBANK

area Area is the amount of surface space inside a flat shape. Area is measured in square units, usually mm², cm², m² or km².

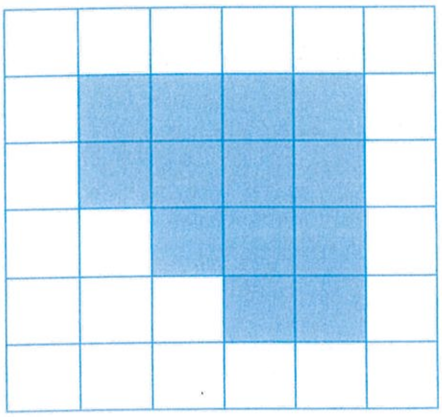
A **square centimetre (cm²)** is the area of a square of length 1 cm.

actual size

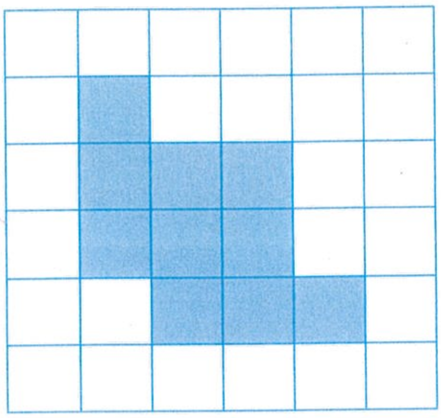
EXAMPLE 1

Find the area of each figure by counting the number of square centimetres.

a



b



SOLUTION

Count the number of shaded squares.

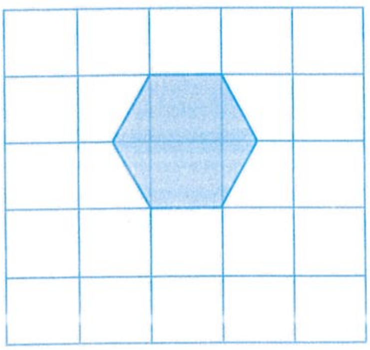
a Area = 13 cm²

b Area = 10 cm²

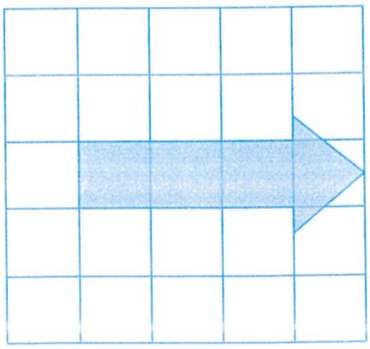
EXAMPLE 2

Find the approximate area of the shapes below.

a



b



SOLUTION

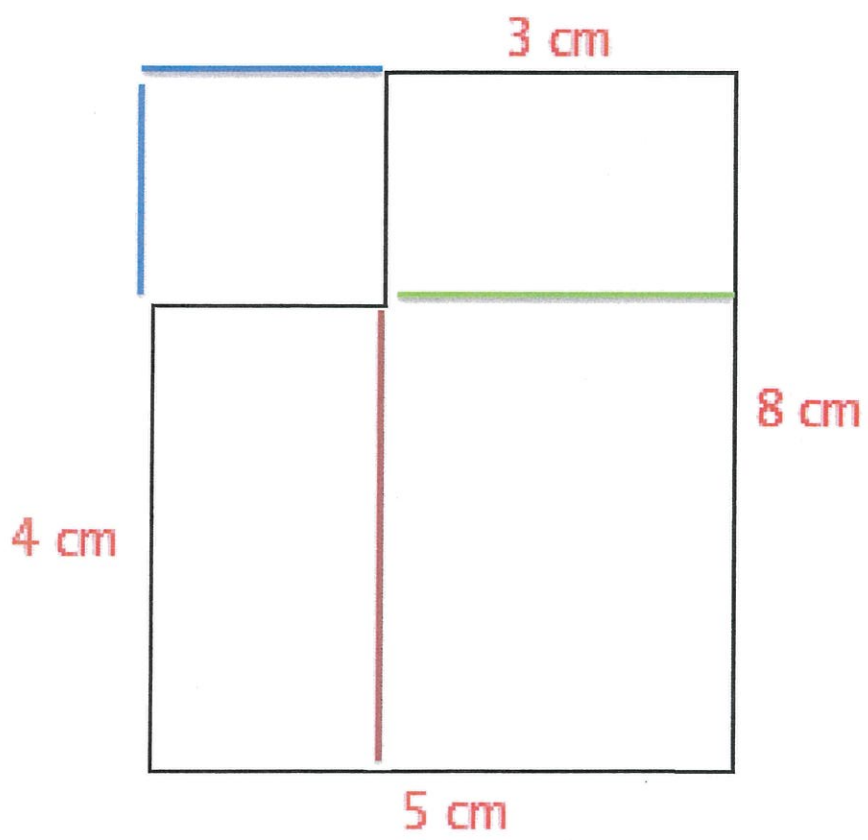
Count whole squares and approximate the rest.

a Approximate area = 3 cm²

b Approximate area = 4 cm²

Measurement

Area



Method 1

Area = 4x2 + 3x8

= 8 + 24

= 32 cm^2

Method 2

Area = 3x4 + 4x5

= 12 + 20

= 32 cm^2

Method 3

Area = 5x8 – 4x2

= 40 – 8

= 32cm^2

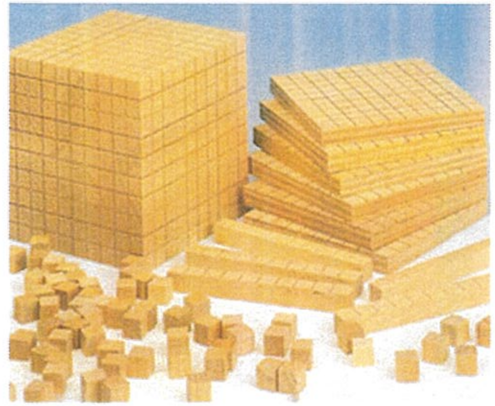
Level 9

Calculates the areas of composite shapes

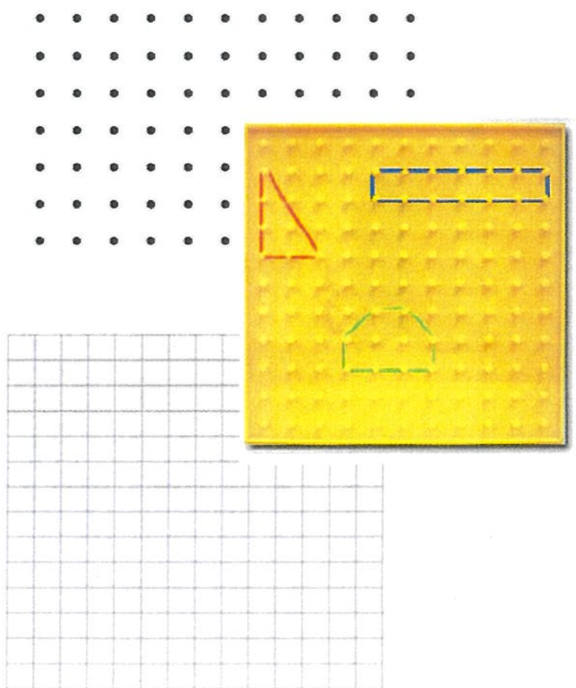
Measurement

Perimeter and Area relationship

Two shapes with the same perimeter but different areas



Two shapes with the same area but different perimeters

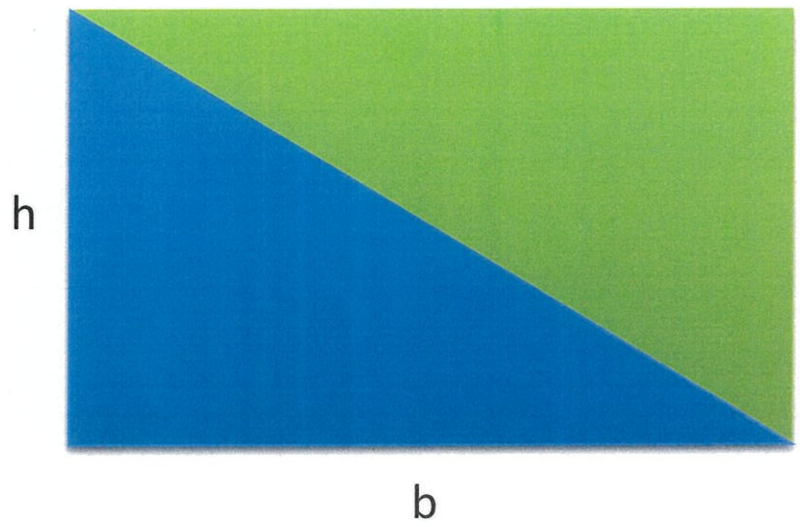


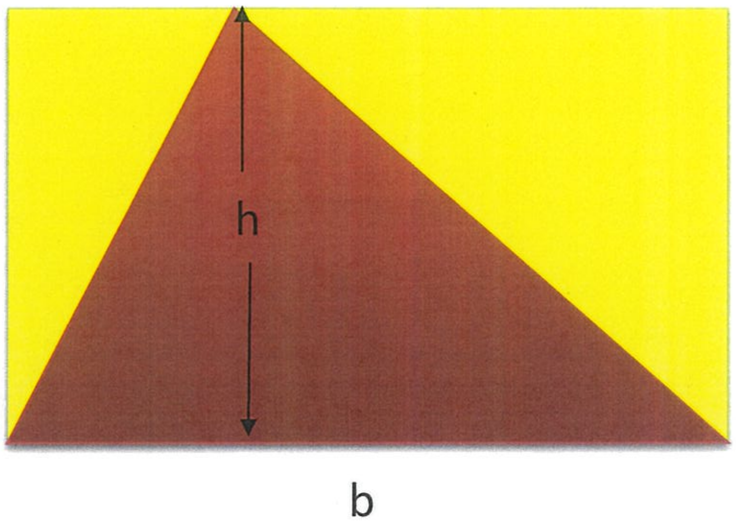
Make a shape – try to change it to a shape that the area decreases but the perimeter increases

Measurement

Triangles

Area of a triangle is $A = \frac{1}{2}bh$





Level 7
Establishes the formulas for areas of rectangles, triangles and parallelograms and uses these in problem solving

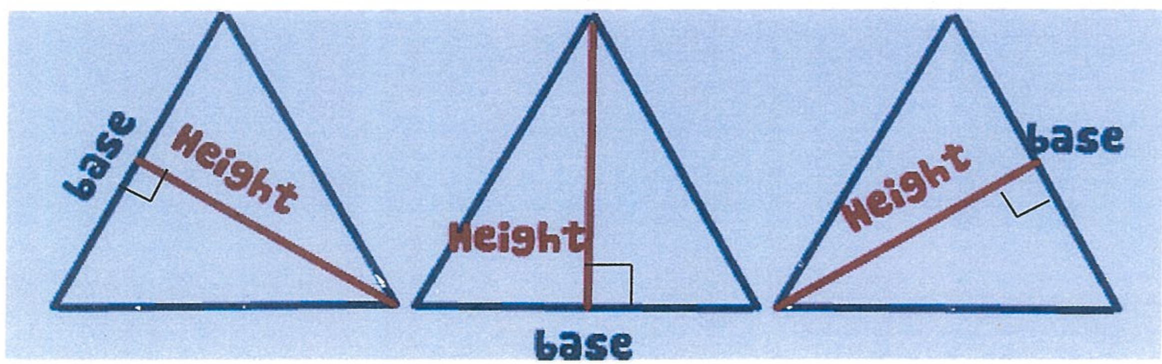
Measurement

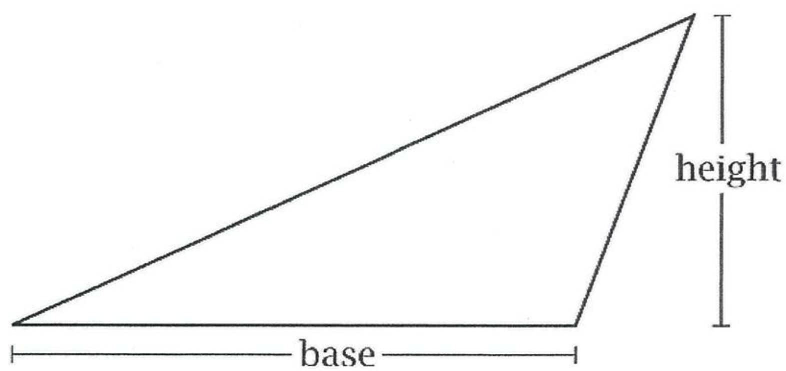
Height and base

Failure to conceptualise the meaning of height and base in 2 dimensional figures

- Ask the question “What happens when we turn the triangle around and thus choose a different height and base?”

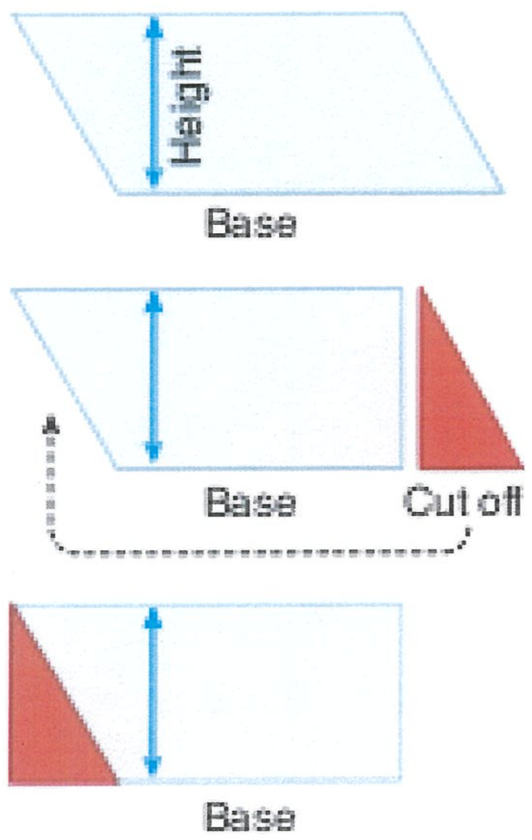
The height is always perpendicular (at a right angle) to the base





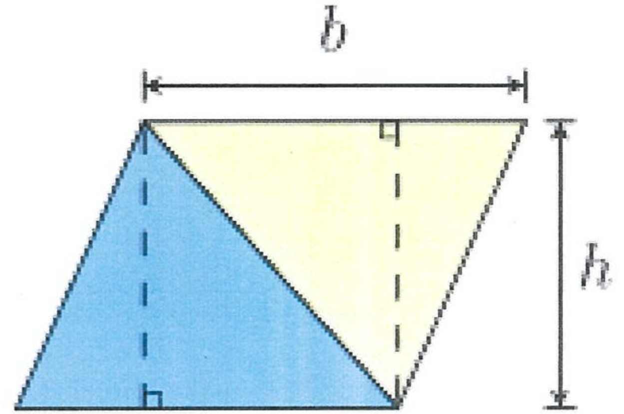
Measurement

Parallelogram



Transforming a parallelogram into a rectangle

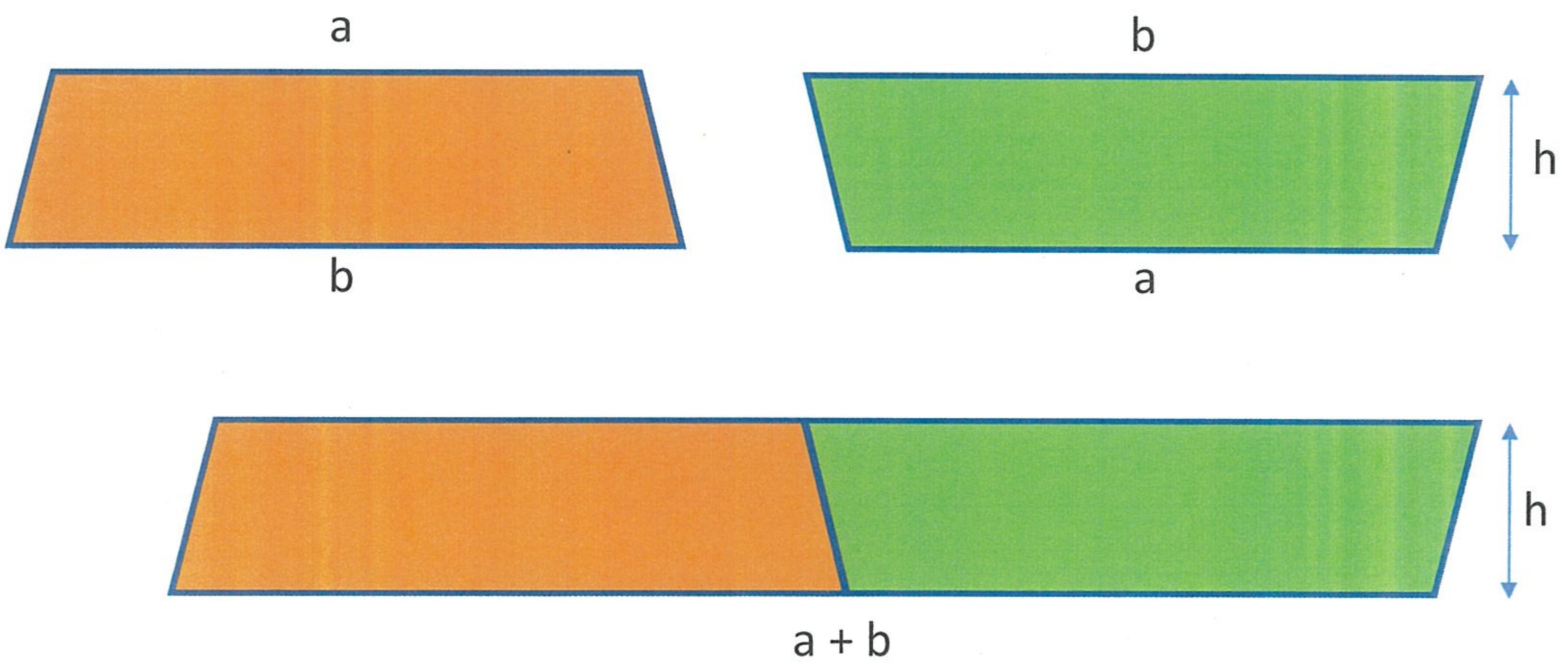
Two triangles will always form a parallelogram with the same base and height.



Area = bh

Measurement

Trapezium



Two congruent trapezoids always make a parallelogram which helps explain the formula below

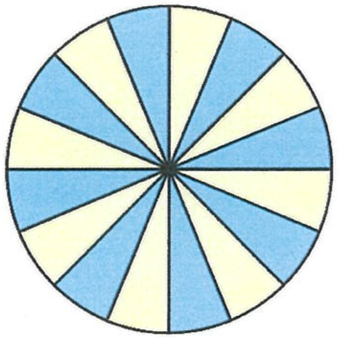
$$A = \frac{1}{2}\text{height} \times (a + b)$$

Level 8

Find perimeters and areas of parallelograms, trapeziums, rhombuses and kites

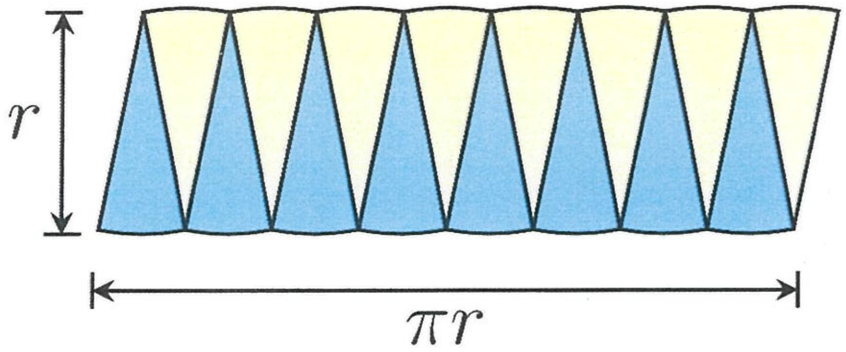
Measurement

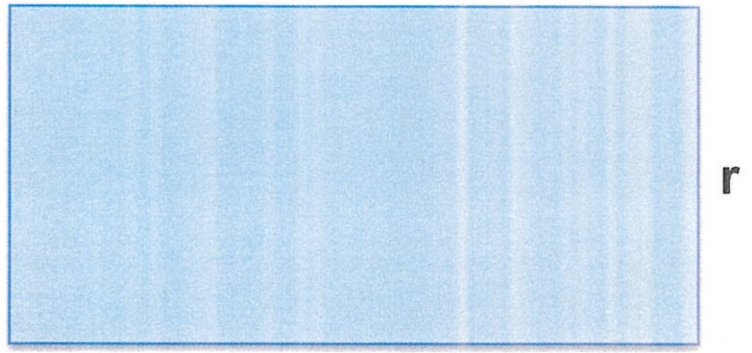
Circles



C= $2\pi r$

8 sectors can form a near parallelogram





πr

Area = $\pi r \times r$

$= \pi r^2$

Triangles

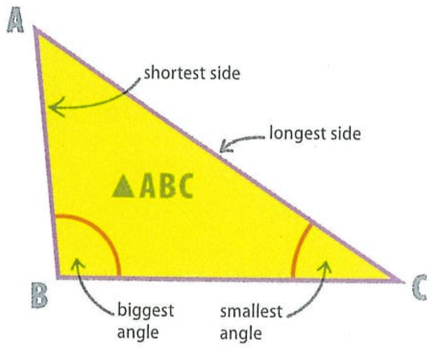
A TRIANGLE IS A SHAPE FORMED WHEN THREE STRAIGHT LINES MEET.

SEE ALSO
❮ 84–85 Angles
❮ 86–87 Straight lines
Constructing triangles 118–119 ❯
Polygons 134–137 ❯

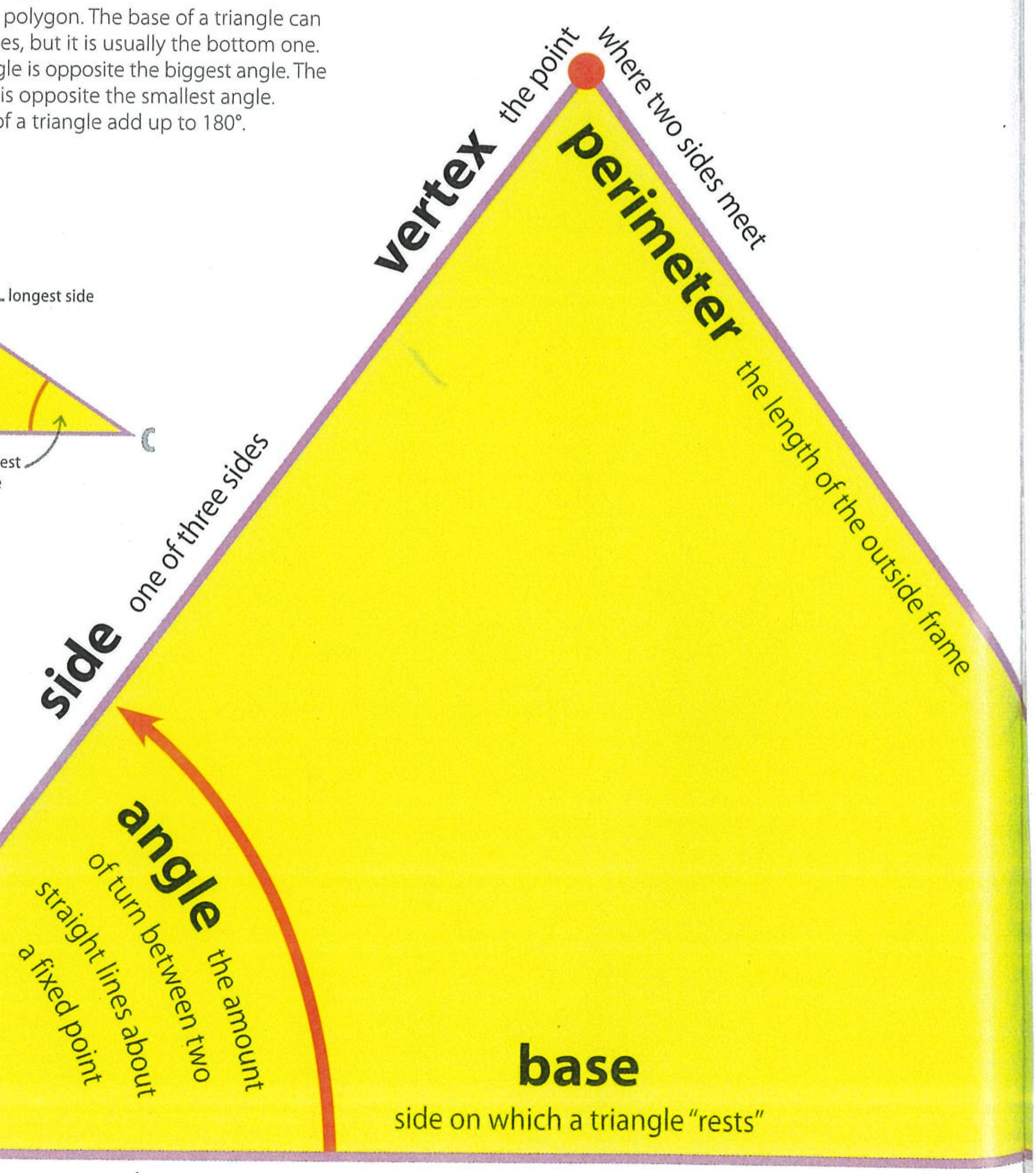
A triangle has three sides and three interior angles. A vertex (plural vertices) is the point where two sides of a triangle meet. A triangle has three vertices.

Introducing triangles

A triangle is a three-sided polygon. The base of a triangle can be any one of its three sides, but it is usually the bottom one. The longest side of a triangle is opposite the biggest angle. The shortest side of a triangle is opposite the smallest angle. The three interior angles of a triangle add up to 180°.

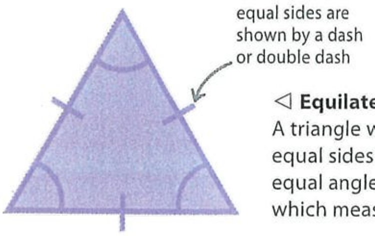


△ Labelling a triangle
A capital letter is used to identify each vertex. A triangle with vertices A, B, and C is known as △ABC. The symbol "△" can be used to represent the word triangle.

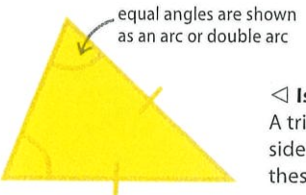


Types of triangles

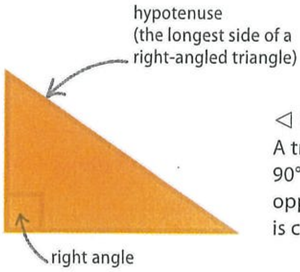
There are several types of triangle, each with specific features, or properties. A triangle is classified according to the length of its sides or the size of its angles.



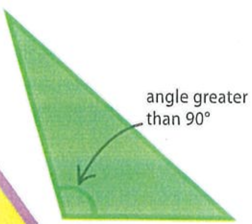
◁ **Equilateral triangle**
A triangle with three equal sides and three equal angles, each of which measures 60°.



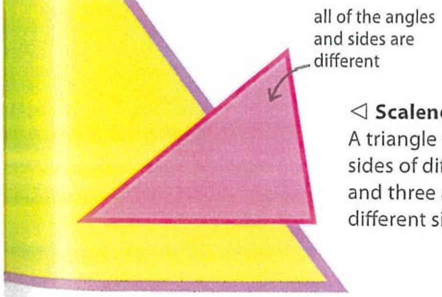
◁ **Isosceles triangle**
A triangle with two equal sides. The angles opposite these sides are also equal.



◁ **Right-angled triangle**
A triangle with an angle of 90° (a right angle). The side opposite the right angle is called the hypotenuse.



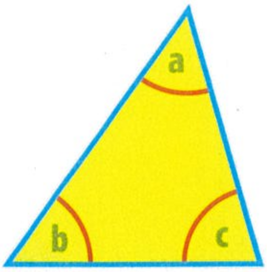
◁ **Obtuse triangle**
A triangle with one angle that measures more than 90°.



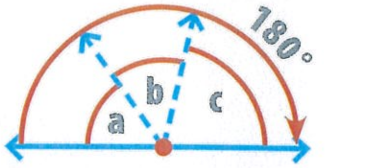
◁ **Scalene triangle**
A triangle with three sides of different length, and three angles of different size.

Interior angles of a triangle

A triangle has three interior angles at the points where each pair of sides meets. These angles always add up to 180°. If rearranged and placed together the angles make up a straight line, which always measures 180°.



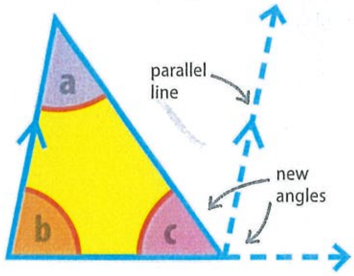
$$a + b + c = 180°$$



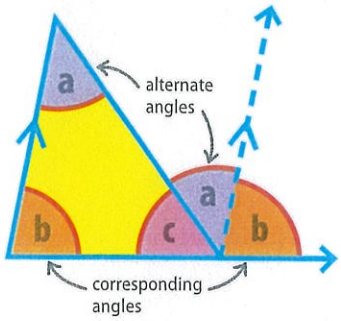
Proving that the sum of a triangle's angles is 180°

Adding a parallel line produces two types of relationships between angles that help prove that the interior sum of a triangle is 180°.

Draw a triangle, then add a line parallel to one side of the triangle, starting at its base, to create two new angles.



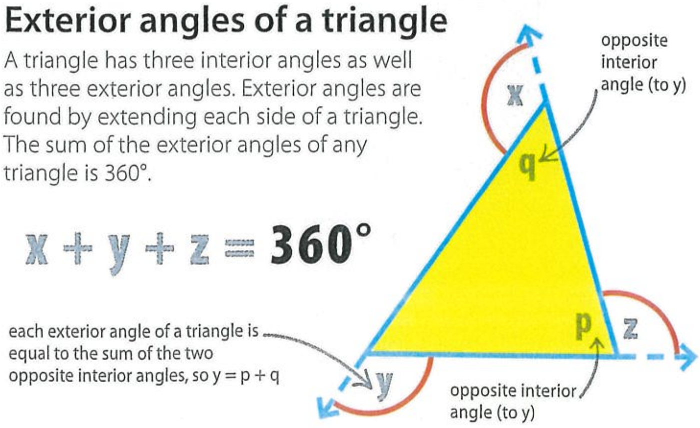
Corresponding angles are equal and alternate angles are equal; angles c, a, and b sit on a straight line so add up to 180°.



Exterior angles of a triangle

A triangle has three interior angles as well as three exterior angles. Exterior angles are found by extending each side of a triangle. The sum of the exterior angles of any triangle is 360°.

$$x + y + z = 360°$$



Area of a triangle

AREA IS THE COMPLETE SPACE INSIDE A TRIANGLE.

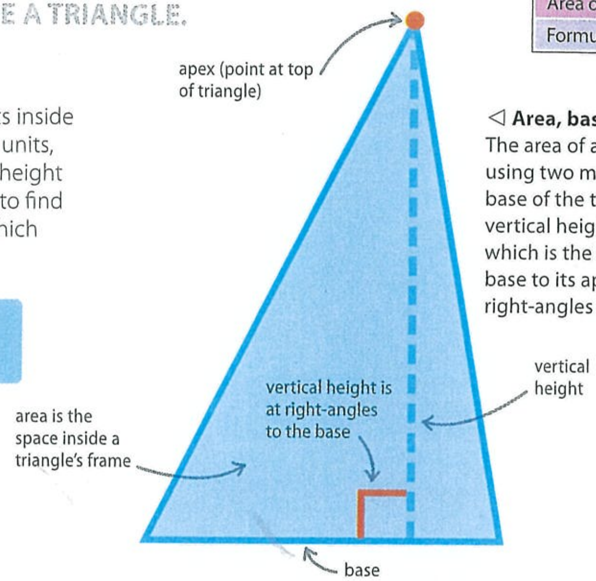
SEE ALSO
❮ 116–117 Triangles
Area of a circle 142–143 ❯
Formulas 177–179 ❯

What is area?

The area of a shape is the amount of space that fits inside its outline, or perimeter. It is measured in squared units, such as cm^2. If the length of the base and vertical height of a triangle are known, these values can be used to find the area of the triangle, using a simple formula, which is shown below.

$$\text{area} = \frac{1}{2} \times \text{base} \times \text{vertical height}$$

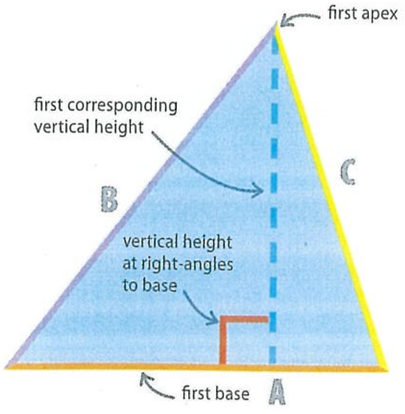
this is the formula for finding the area of a triangle



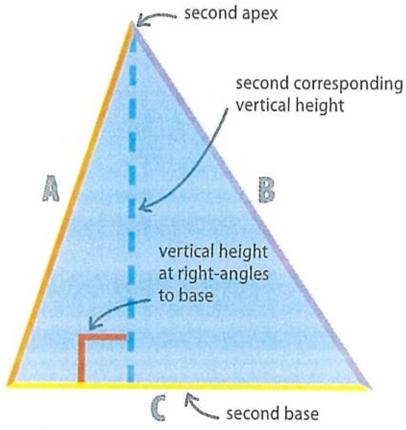
◁ **Area, base, and height**
The area of a triangle is found using two measurements: the base of the triangle and the vertical height of the triangle, which is the distance from its base to its apex, measured at right-angles to the base.

Base and vertical height

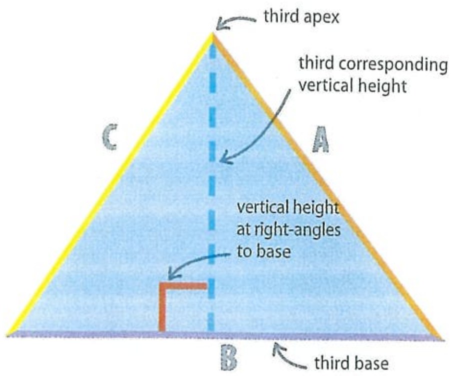
Finding the area of a triangle requires two measurements: the base and the vertical height. The side on which a triangle "sits" is called the base. The vertical height is a line formed at right-angles to the base from the apex. Any one of the three sides of a triangle can act as the base in the area formula.



△ **First base**
The area of the triangle can be found using the orange side (A) as the "base" needed for the formula. The corresponding vertical height is the distance from the base of the triangle to its apex (highest point).



△ **Second base**
Any one of the triangle's three sides can act as its base. Here the triangle is rotated so that the green side (C) is its base. The corresponding vertical height is the distance from the base to the apex.

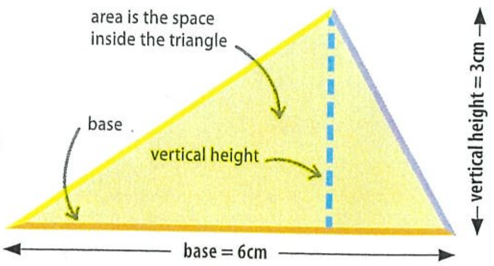


△ **Third base**
The triangle is rotated again, so that the purple side (B) is its base. The corresponding vertical height is the distance from the base to the apex. The area of the triangle is the same, whichever side is used as the base in the formula.

Finding the area of a triangle

To calculate the area of a triangle, substitute the given values for the base and vertical height into the formula. Then work through the multiplication shown by the formula (½ × base × vertical height).

▷ **An acute-angled triangle**
The base of this triangle is 6cm and its vertical height is 3cm. Find the area of the triangle using the formula.



First, write down the formula for the area of a triangle.

$$\text{area} = \frac{1}{2} \times \text{base} \times \text{vertical height}$$

Then, substitute the lengths that are known into the formula.

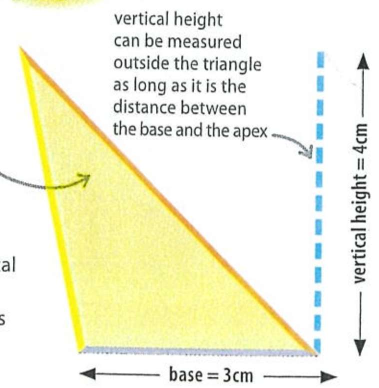
$$\text{area} = \frac{1}{2} \times 6 \times 3$$

Work through the multiplication in the formula to find the answer. In this example, ½ × 6 × 3 = 9. Add the units of area to the answer, here cm².

area is measured in squared units

$$\text{area} = 9\text{cm}^2$$

▷ **An obtuse-angled triangle**
The base of this triangle is 3cm and its vertical height is 4cm. Find the area of the triangle using the formula. The formula and the steps are the same for all types of triangles.



First, write down the formula for the area of a triangle.

$$\text{area} = \frac{1}{2} \times \text{base} \times \text{vertical height}$$

Then, substitute the lengths that are known into the formula.

$$\text{area} = \frac{1}{2} \times 3 \times 4$$

Work through the multiplication to find the answer, and add the appropriate units of area.

½ × 3 × 4 = 6

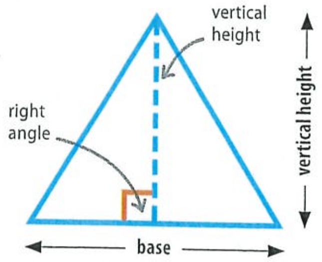
area is measured in squared units

$$\text{area} = 6\text{cm}^2$$

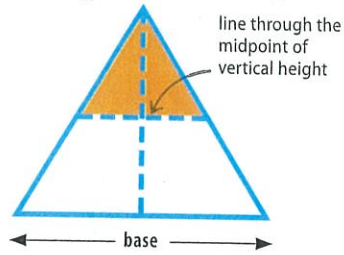
LOOKING CLOSER

Why the formula works

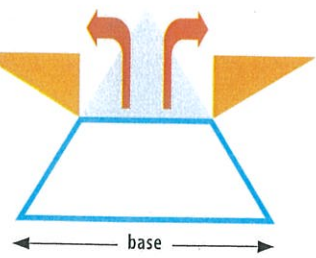
By adjusting the shape of a triangle, it can be converted into a rectangle. This process makes the formula for a triangle easier to understand.



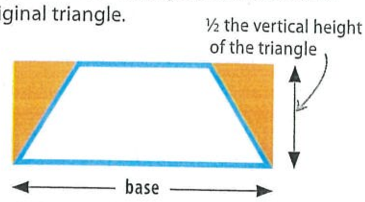
Draw any triangle and label its base and vertical height.



Draw a line through the midpoint of the vertical height, which is parallel to the base.



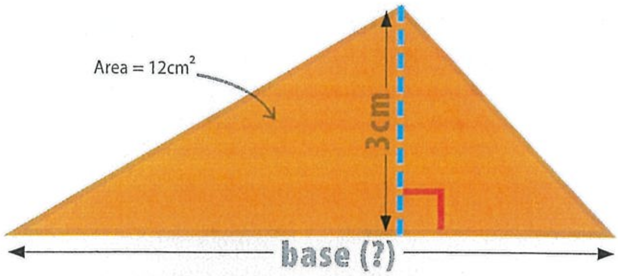
This creates two new triangles. These can be rotated around the triangle to form a rectangle. This has exactly the same area as the original triangle.



The original triangle's area is found using the formula for the area of a rectangle (b × h). Both shapes have the same base; the rectangle's height is ½ the height of the triangle. This gives the area of the triangle formula: ½ × base × vertical height.

Finding the base of a triangle using the area and height

The formula for the area of a triangle can also be used to find the length of the base, if the area and height are known. Given the area and height of the triangle, the formula needs to be rearranged to find the length of the triangle's base.



First, write down the formula for the area of a triangle. The formula states that the area of a triangle is equal to ½ multiplied by the length of the base, multiplied by the height.

$$\text{area} = \frac{1}{2} \times \text{base} \times \text{height}$$

Substitute the known values into the formula. Here the values of the area (12cm²) and the height (3cm) are known.

$$12 = \frac{1}{2} \times \text{base} \times 3$$

Simplify the formula as far as possible, by multiplying the ½ by the height. This answer is 1.5.

½ × 3 = 1.5

$$12 = 1.5 \times \text{base}$$

base is unknown

Make the base the subject of the formula by rearranging it. In this example both sides are divided by 1.5.

$$\frac{12}{1.5} = \text{base}$$

as base was multiplied by 1.5, divide this side by 1.5 to cancel out the 1.5s and leave base on its own on this side

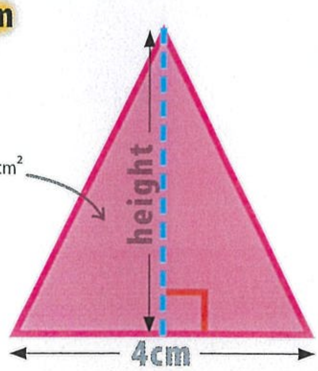
as the other side has been divided by 1.5, this side must also be divided by 1.5

Work out the final answer by dividing 12 (area) by 1.5. In this example, the answer is 8cm.

$$\text{base} = \mathbf{8cm}$$

Finding the vertical height of a triangle using the area and base

The formula for area of a triangle can also be used to find its height, if the area and base are known. Given the area and the length of the base of the triangle, the formula needs to be rearranged to find the height of the triangle.



First, write down the formula. This shows that the area of a triangle equals ½ multiplied by its base, multiplied by its height.

$$\text{area} = \frac{1}{2} \times \text{base} \times \text{height}$$

Substitute the known values into the formula. Here the values of the area (8cm²) and the base (4cm) are known.

$$8 = \frac{1}{2} \times 4 \times \text{height}$$

height is unknown

Simplify the equation as far as possible, by multiplying the ½ by the base. In this example, the answer is 2.

½ × 4 = 2

$$8 = 2 \times \text{height}$$

Make the height the subject of the formula by rearranging it. In this example both sides are divided by 2.

$$\frac{8}{2} = \text{height}$$

this side must be divided by 2 to cancel out the 2s and leave height on its own on this side

as the other side has been divided by 2, this side must also be divided by 2

Work out the final answer by dividing 8 (the area) by 2 (½ the base). In this example the answer is 4cm.

$$\text{height} = \mathbf{4cm}$$

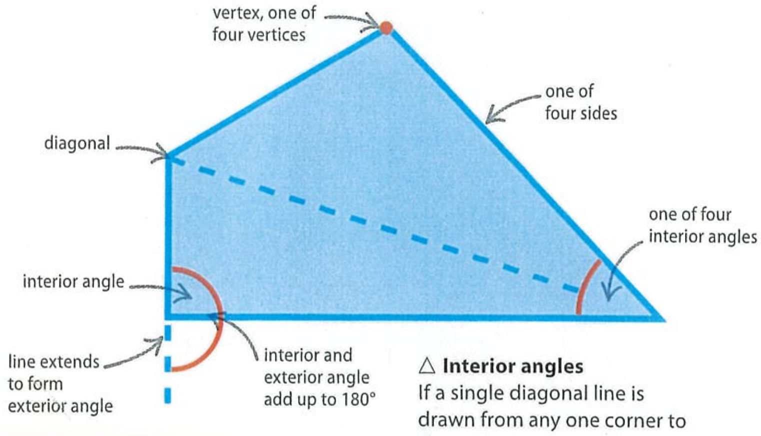
Quadrilaterals

SEE ALSO
❮ 84–85 Angles
❮ 86–87 Straight lines
Polygons 134–137 ❯

A QUADRILATERAL IS A FOUR-SIDED POLYGON. "QUAD" MEANS FOUR AND "LATERAL" MEANS SIDE.

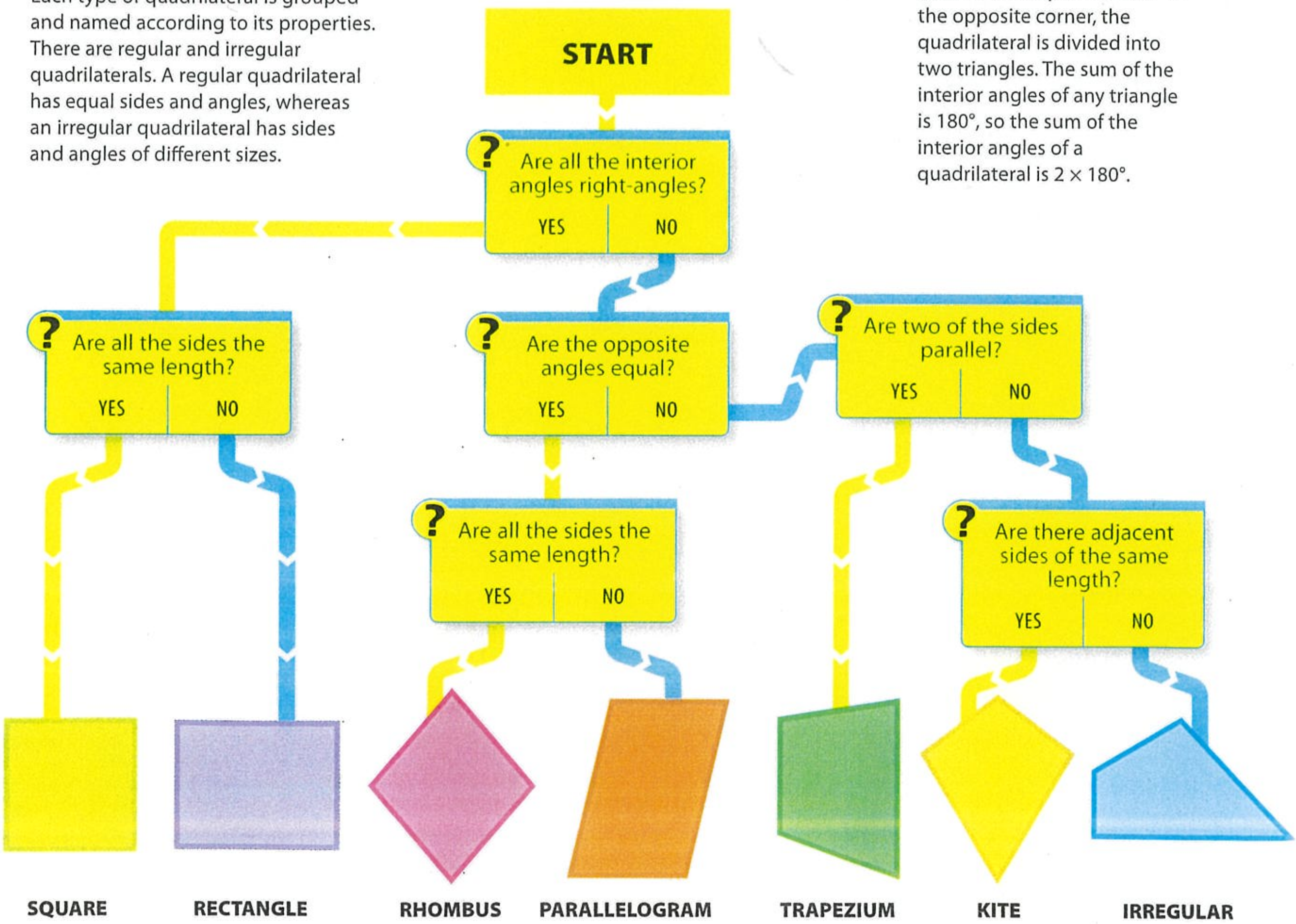
Introducing quadrilaterals

A quadrilateral is a two-dimensional shape with four straight sides, four vertices (points where the sides meet), and four interior angles. The interior angles of a quadrilateral always add up to 360°. An exterior angle and its corresponding interior angle always add up to 180° because they form a straight line. There are several types of quadrilaterals, each with different properties.



△ **Interior angles**
If a single diagonal line is drawn from any one corner to the opposite corner, the quadrilateral is divided into two triangles. The sum of the interior angles of any triangle is 180°, so the sum of the interior angles of a quadrilateral is 2 × 180°.

▽ **Types of quadrilaterals**
Each type of quadrilateral is grouped and named according to its properties. There are regular and irregular quadrilaterals. A regular quadrilateral has equal sides and angles, whereas an irregular quadrilateral has sides and angles of different sizes.

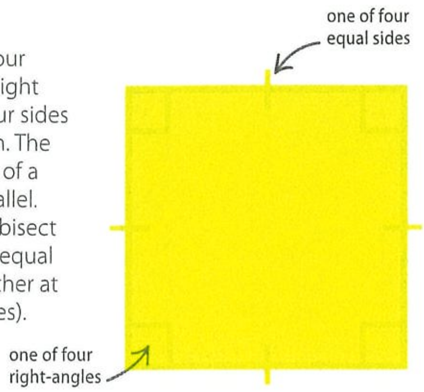


PROPERTIES OF QUADRILATERALS

Each type of quadrilateral has its own name and a number of unique properties. Knowing just some of the properties of a shape can help distinguish one type of quadrilateral from another. Six of the more common quadrilaterals are shown below with their respective properties.

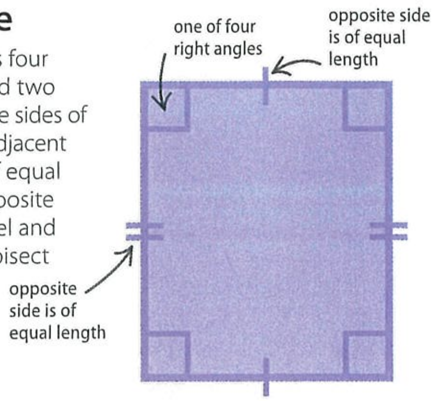
Square

A square has four equal angles (right angles) and four sides of equal length. The opposite sides of a square are parallel. The diagonals bisect – cut into two equal parts – each other at 90° (right angles).



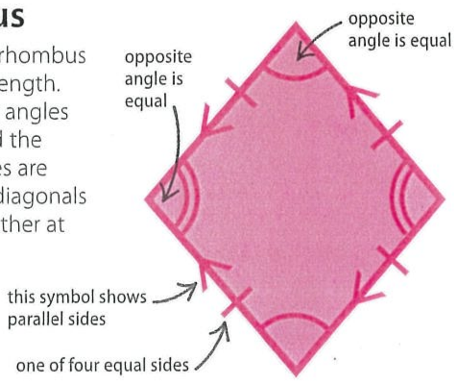
Rectangle

A rectangle has four right angles and two pairs of opposite sides of equal length. Adjacent sides are not of equal length. The opposite sides are parallel and the diagonals bisect each other.



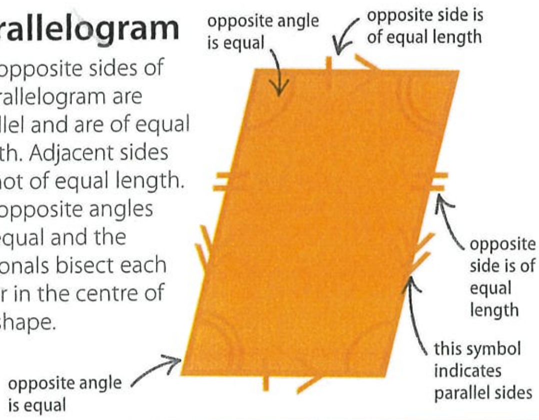
Rhombus

All sides of a rhombus are of equal length. The opposite angles are equal and the opposite sides are parallel. The diagonals bisect each other at right angles.



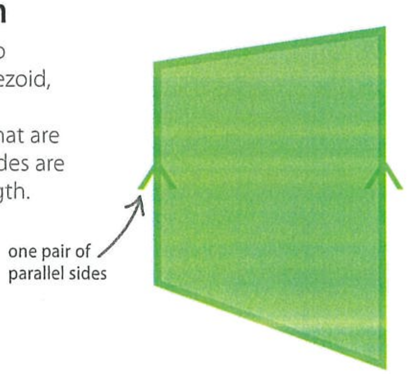
Parallelogram

The opposite sides of a parallelogram are parallel and are of equal length. Adjacent sides are not of equal length. The opposite angles are equal and the diagonals bisect each other in the centre of the shape.



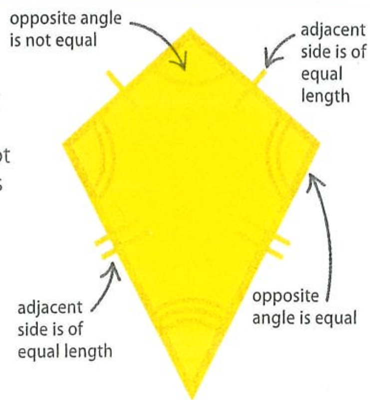
Trapezium

A trapezium, also known as a trapezoid, has one pair of opposite sides that are parallel. These sides are not equal in length.



Kite

A kite has two pairs of adjacent sides that are equal in length. Opposite sides are not of equal length. It has one pair of opposite angles that are equal and another pair of angles of different values.

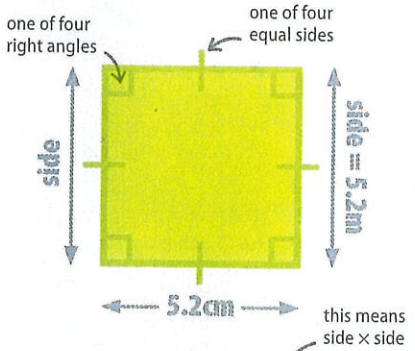


FINDING THE AREA OF QUADRILATERALS

Area is the space inside the frame of a two-dimensional shape. Area is measured in square units, for example, cm². Formulas are used to calculate the areas of many types of shapes. Each type of quadrilateral has a unique formula for calculating its area.

Finding the area of a square

The area of a square is found by multiplying its length by its width. As its length and width are equal in size, the formula is the square of a side.



this means side × side

$$\text{area} = \text{side}^2$$

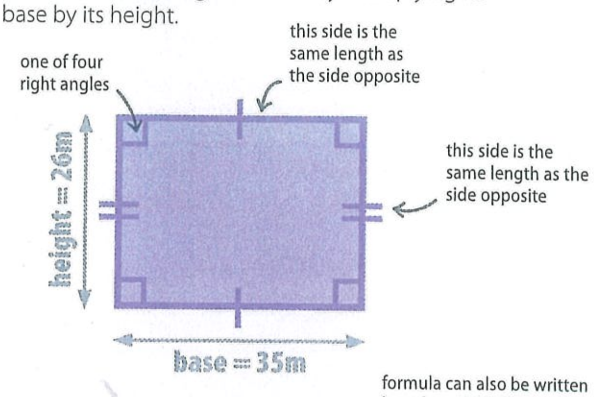
units of area, square centimetres

$$5.2 \times 5.2 = 27.04\text{cm}^2$$

△ **Multiply sides**
In this example, each of the four sides measures 5.2cm. To find the area of this square, multiply 5.2 by 5.2.

Finding the area of a rectangle

The area of a rectangle is found by multiplying its base by its height.



formula can also be written length × width (l × w)

$$\text{area} = \text{base} \times \text{height}$$

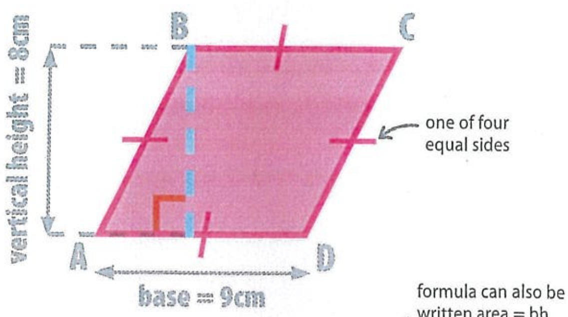
units of area, square metres

$$35 \times 26 = 910\text{m}^2$$

△ **Multiply base by height**
The height (or width) of this rectangle is 26m, and its base (or length) measures 35m. Multiply these two measurements together to find the area.

Finding the area of a rhombus

The area of a rhombus is found by multiplying the length of its base by its vertical height. The vertical height, also known as the perpendicular height, is the vertical distance from the top (vertex) of a shape to the base opposite. The vertical height is at right angles to the base.



▷ **Vertical height**
Finding the area of a rhombus depends on knowing its vertical height. In this example, the vertical height measures 8cm and its base is 9cm.

formula can also be written area = bh

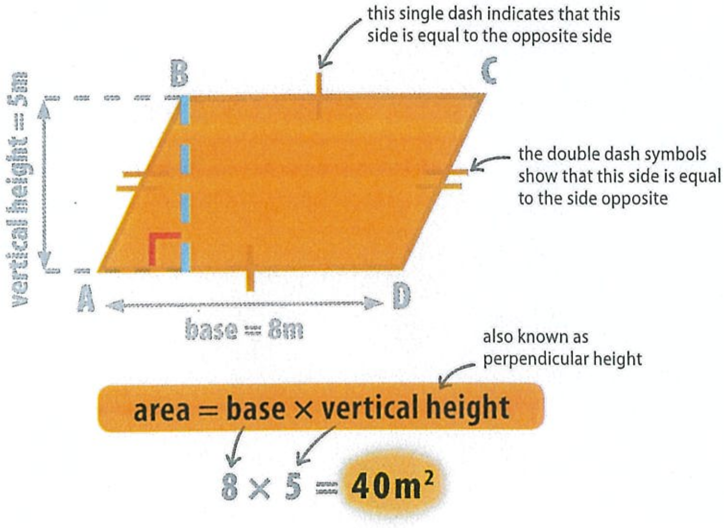
$$\text{area} = \text{base} \times \text{vertical height}$$

$$9 \times 8 = 72\text{cm}^2$$

Finding the area of a parallelogram

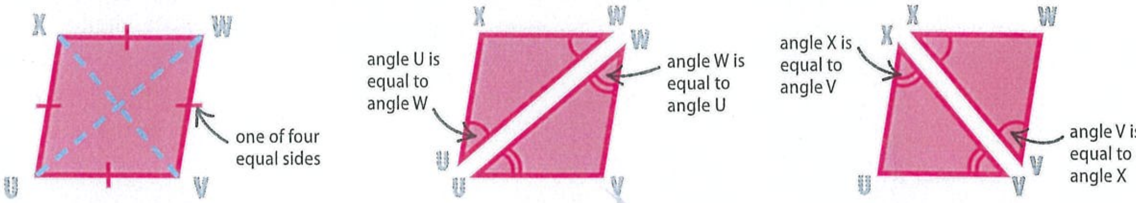
Like the area of a rhombus, the area of a parallelogram is found by multiplying the length of its base by its vertical height.

▷ **Multiply base by vertical height**
It is important to remember that the slanted side, AB, is not the vertical height. This formula only works if the vertical height is used.



Proving the opposite angles of a rhombus are equal

Creating two pairs of isosceles triangles by dividing a rhombus along two diagonals helps prove that the opposite angles of a rhombus are equal. An isosceles triangle has two equal sides and two equal angles.



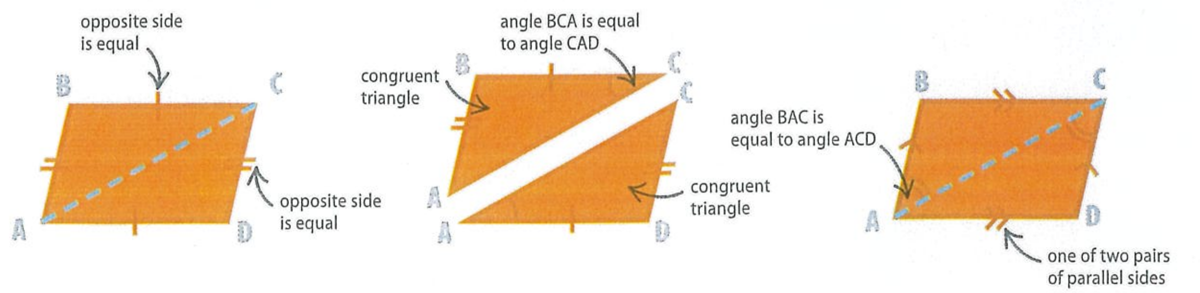
All the sides of a rhombus are equal in length. To show this a dash is used on each side.

▷ **Divide the rhombus** along the diagonals to create two isosceles triangles. Each triangle has a pair of equal angles.

▷ **Dividing along the other diagonal** creates another pair of isosceles triangles.

Proving the opposite sides of a parallelogram are parallel

Creating a pair of congruent triangles by dividing a parallelogram along two diagonals helps prove that the opposite sides of a parallelogram are parallel. Congruent triangles are the same size and shape.



Opposites sides of a parallelogram are equal in length. To show this a dash and a double dash are used.

▷ **The triangles ABC and ADC** are congruent. Angle BCA = CAD, and as these are alternate angles, BC is parallel to AD.

▷ **The triangles are congruent,** so angle BAC = ACD; as these are alternate angles, DC is parallel to AB.

Polygons

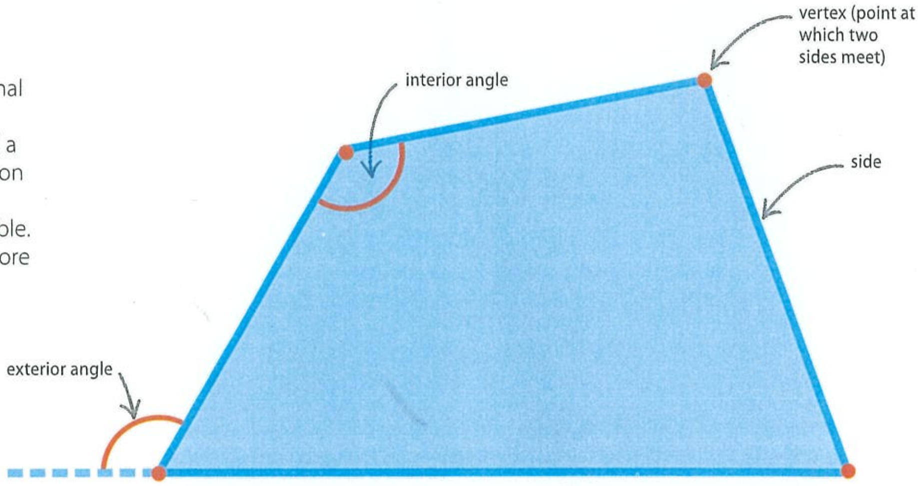
A CLOSED TWO-DIMENSIONAL SHAPE OF THREE OR MORE SIDES.

SEE ALSO
❮ 84–85 Angles
❮ 116–117 Triangles
❮ 120–121 Congruent triangles
❮ 130–133 Quadrilaterals

Polygons range from simple three-sided triangles and four-sided squares to more complicated shapes such as trapezoids and dodecagons. Polygons are named according to the number of sides and angles they have.

What is a polygon?

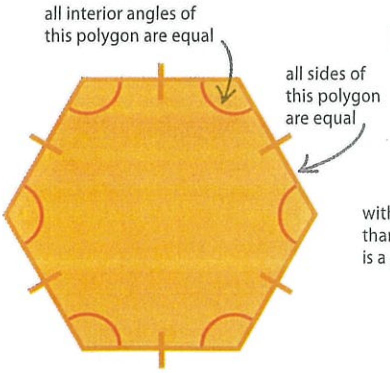
A polygon is a closed two-dimensional shape formed by straight lines that connect end to end at a point called a vertex. The interior angles of a polygon are usually smaller than the exterior angles, although the reverse is possible. Polygons with an interior angle of more than 180° are called re-entrant.



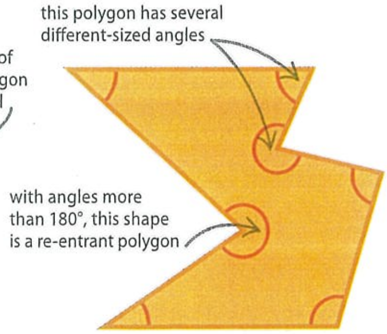
▷ **Parts of a polygon**
Regardless of shape, all polygons are made up the same parts – sides, vertices (connecting points), and interior and exterior angles.

Describing polygons

There are several ways to describe polygons. One is by the regularity or irregularity of their sides and angles. A polygon is regular when all of its sides and angles are equal. An irregular polygon has at least two sides or two angles that are different.



△ **Regular**
All the sides and all the angles of regular polygons are equal. This hexagon has six equal sides and six equal angles, making it regular.

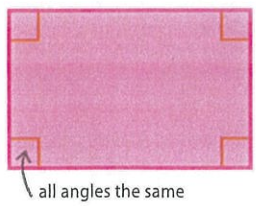


△ **Irregular**
In an irregular polygon, all the sides and angles are not the same. This heptagon has many different-sized angles, making it irregular.

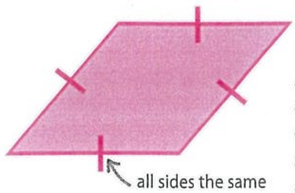
LOOKING CLOSER

Equal angles or equal sides?

All the angles and all the sides of a regular polygon are equal – in other words, the polygon is both equiangular and equilateral. In certain polygons, only the angles (equiangular) or only the sides (equilateral) are equal.



◁ **Equiangular**
A rectangle is an equiangular quadrilateral. Its angles are all equal, but not all its sides are equal.



◁ **Equilateral**
A rhombus is an equilateral quadrilateral. All its sides are equal, but all its angles are not.

Naming polygons

Regardless of whether a polygon is regular or irregular, the number of sides it has always equals the number of its angles. This number is used in naming both kinds of polygons. For example, a polygon with six sides and angles is called a hexagon because "hex" is the prefix used to mean six. If all of its sides and angles are equal, it is known as a regular hexagon; if not, it is called an irregular hexagon.

Triangle

3
Sides and angles

Quadrilateral

4
Sides and angles

Pentagon

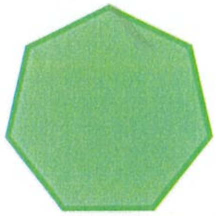
5
Sides and angles

Hexagon

6
Sides and angles

Heptagon

7
Sides and angles

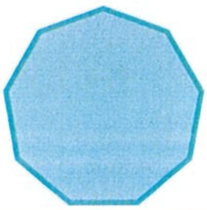


Octagon

8
Sides and angles

Nonagon

9
Sides and angles

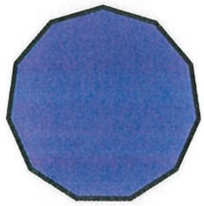


Decagon

10
Sides and angles

Hendecagon

11
Sides and angles

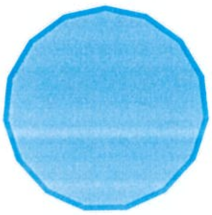


Dodecagon

12
Sides and angles

Pentadecagon

15
Sides and angles



Icosagon

20
Sides and angles

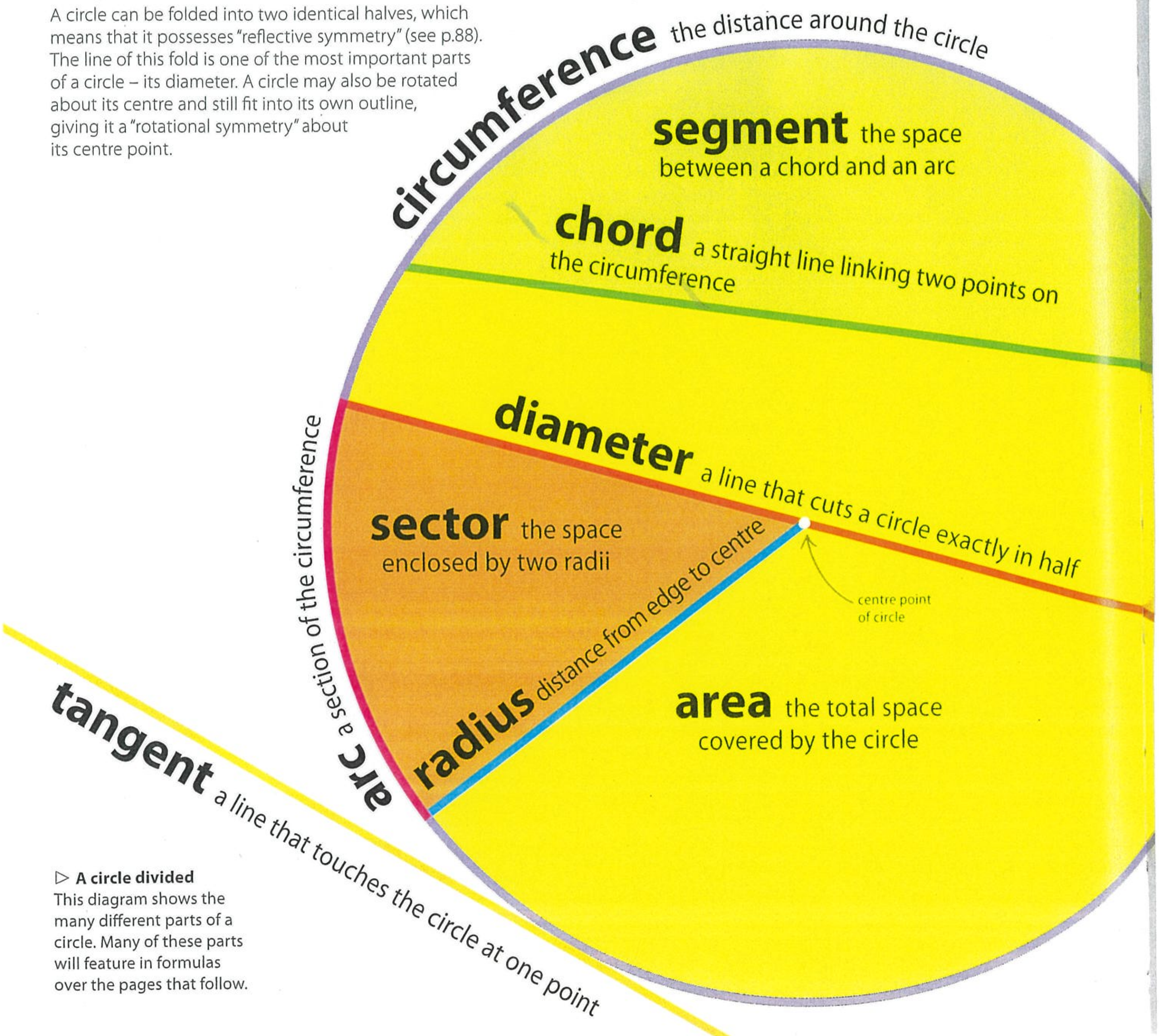
Circles

A CIRCLE IS A CLOSED CURVED LINE SURROUNDING A CENTRE POINT. EVERY POINT OF THIS CURVED LINE IS OF EQUAL DISTANCE FROM THE CENTRE POINT.

SEE ALSO
❮ 82–83 Tools in geometry
Circumference and diameter 140–141 ❯
Area of a circle 142–143 ❯

Properties of a circle

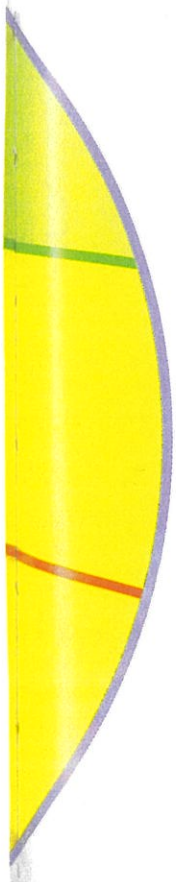
A circle can be folded into two identical halves, which means that it possesses "reflective symmetry" (see p.88). The line of this fold is one of the most important parts of a circle – its diameter. A circle may also be rotated about its centre and still fit into its own outline, giving it a "rotational symmetry" about its centre point.



▷ **A circle divided**
This diagram shows the many different parts of a circle. Many of these parts will feature in formulas over the pages that follow.

Parts of a circle

A circle can be measured and divided in various ways. Each of these has a specific name and character, and they are all shown below.



Radius
Any straight line from the centre of a circle to its circumference. The plural of radius is radii.

Diameter
Any straight line that passes through the centre from one side of a circle to the other.

Chord
Any straight line linking two points on a circle's circumference, but not passing through its centre.

Segment
The smaller of the two parts of a circle created when divided by a chord.

Circumference
The total length of the outside edge (perimeter) of a circle.

Arc
Any section of the circumference of a circle.

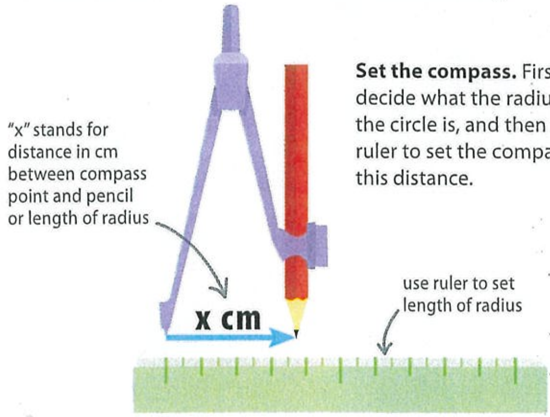
Sector
A "slice" of a circle, similar to the slice of a pie. It is enclosed by two radii and an arc.

Area
The amount of space inside a circle's circumference.

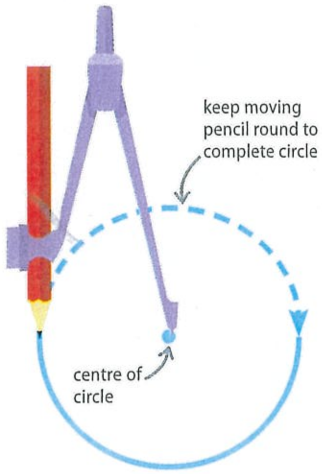
Tangent
A straight line that touches the circle at a single point.

How to draw a circle

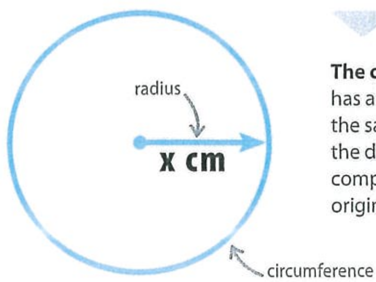
Two instruments are needed to draw a circle – a compass and a pencil. The point of the compass marks the centre of the circle and the distance between the point and the pencil attached to the compass forms the circle's radius. A ruler is needed to measure the radius of the circle correctly.



Set the compass. First, decide what the radius of the circle is, and then use a ruler to set the compass at this distance.



Decide where the centre of the circle is and then hold the point of the compass firmly in this place. Then put the pencil to the paper and move the pencil round to draw the circumference of the circle.



The completed circle has a radius that is the same length as the distance that the compass was originally set to.

Circumference and diameter

SEE ALSO
❮ 56–59 Ratio and proportion
❮ 104–105 Enlargements
❮ 138–139 Circles
Area of a circle 142–143 ❯

THE DISTANCE AROUND THE EDGE OF A CIRCLE IS CALLED THE CIRCUMFERENCE; THE DISTANCE ACROSS THE MIDDLE IS THE DIAMETER.

All circles are similar because they have exactly the same shape. This means that all their measurements, including the circumference and the diameter, are in proportion to each other.

The number pi

The ratio between the circumference and diameter of a circle is a number called pi, which is written π. This number is used in many of the formulas associated with circles, including the formulas for the circumference and diameter.

symbol for pi

$$\pi = 3.14$$

value to 2 decimal places

◁ **The value of pi**
The numbers after the decimal point in pi go on for ever and in an unpredictable way. It starts 3.1415926 but is usually given to two decimal places.

Circumference (c)

The circumference is the distance around the edge of a circle. A circle's circumference can be found using the diameter or radius and the number pi. The diameter is always twice the length of the radius.

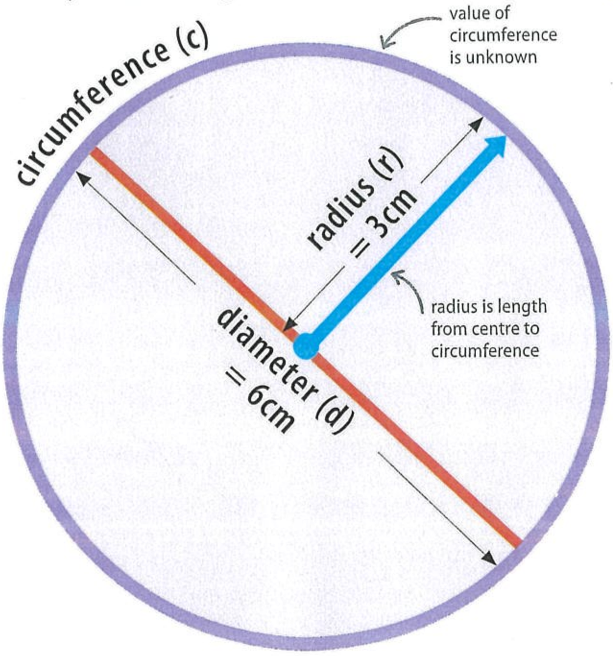
circumference, π is a constant, radius

$$c = 2\pi r$$

circumference, π is a constant, diameter

$$c = \pi d$$

◁ **Formulas**
There are two circumference formulas. One uses diameter and the other uses radius.



△ **Finding the circumference**
The length of a circle's circumference can be found if the length of the diameter is known, in this example the diameter is 6cm long.

The formula for circumference shows that the circumference is equal to pi multiplied by the diameter of the circle.

$$c = \pi d$$

d is the same as $2 \times r$, the formula can also be written $C=2\pi r$

Substitute known values into the formula for circumference. Here, the radius of the circle is known to be 3cm.

$$c = 3.14 \times 6$$

pi is 3.14 to two decimal places

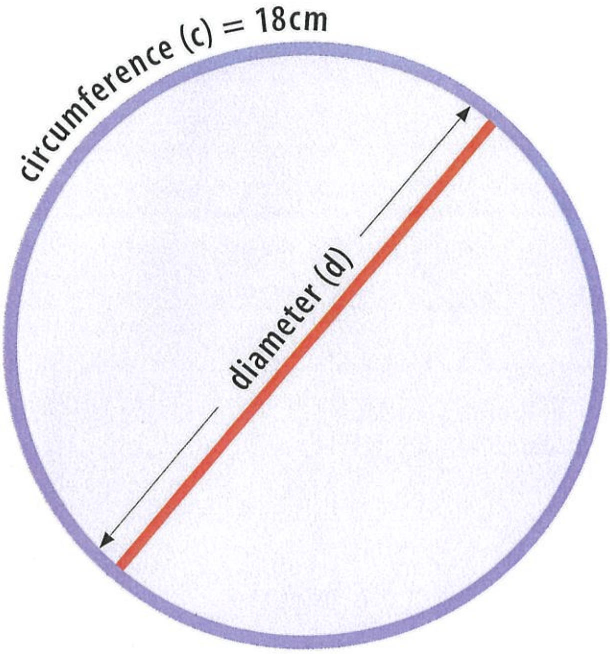
Multiply the numbers to find the length of the circumference. Round the answer to a suitable number of decimal places.

$$c = 18.8\text{cm}$$

18.84 is rounded to one decimal place

Diameter (d)

The diameter is the distance across the middle of a circle. It is twice the length of the radius. A circle's diameter can be found by doubling the length of its radius, or by using its circumference and the number pi in the formula shown below. The formula is a rearranged version of the formula for the circumference of a circle.



△ **Finding the diameter**
This circle has a circumference of 18cm. Its diameter can be found using the formula given above.

The formula for diameter shows that the length of the diameter is equal to the length of the circumference divided by the number pi.

diameter, circumference

$$d = \frac{c}{\pi}$$

π is a constant

Substitute known values into the formula for diameter. In the example shown here, the circumference of the circle is 18cm.

$$d = \frac{18}{\pi}$$

Divide the circumference by the value of pi, 3.14, to find the length of the diameter.

$$d = \frac{18}{3.14}$$

more accurate to use π button on a calculator

Round the answer to a suitable number of decimal places. In this example, the answer is given to two decimal places.

$$d = 5.73\text{cm}$$

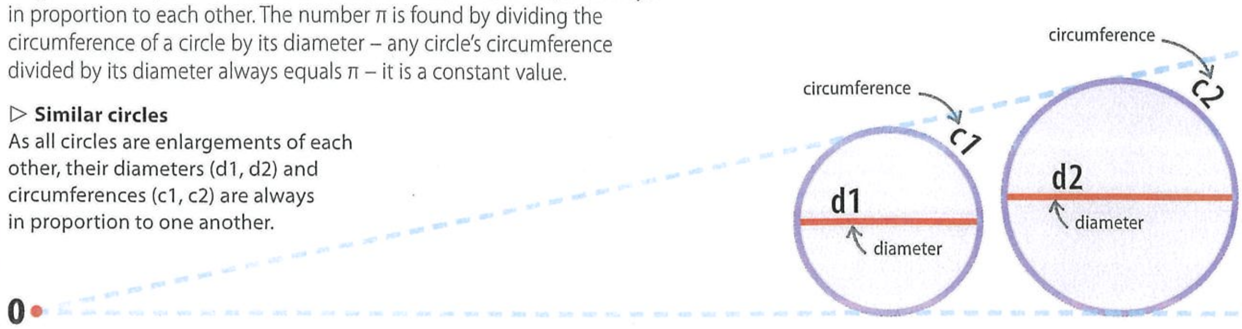
the answer is given to two decimal places

LOOKING CLOSER

Why π ?

All circles are similar to one another. This means that corresponding lengths in circles, such as their diameters and circumferences, are always in proportion to each other. The number π is found by dividing the circumference of a circle by its diameter – any circle's circumference divided by its diameter always equals π – it is a constant value.

▷ **Similar circles**
As all circles are enlargements of each other, their diameters (d1, d2) and circumferences (c1, c2) are always in proportion to one another.



Area of a circle

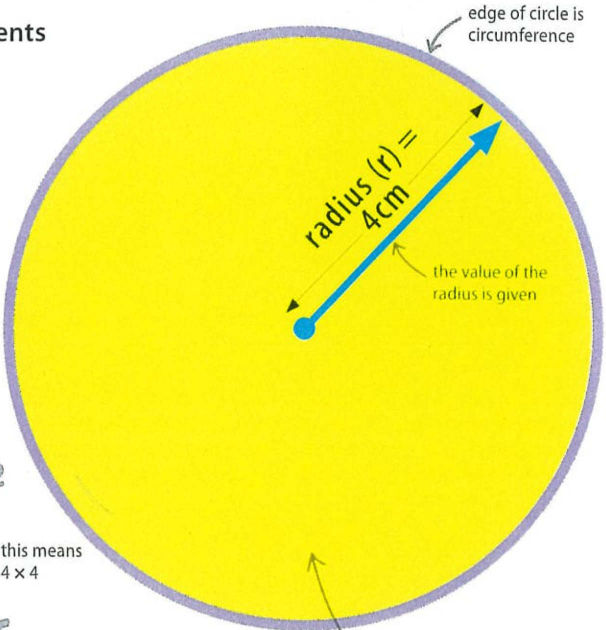
THE AREA OF A CIRCLE IS THE AMOUNT OF SPACE ENCLOSED INSIDE ITS PERIMETER (CIRCUMFERENCE).

SEE ALSO
❮ 138–139 Circles
❮ 140–141 Circumference and diameter
Formulas 177–179 ❯

The area of a circle can be found by using the measurements of either the radius or the diameter of the circle.

Finding the area of a circle

The area of a circle is measured in square units. It can be found using the radius of a circle (r) and the formula shown below. If the diameter is known but the radius is not, the radius can be found by dividing the diameter by 2.



In the formula for the area of a circle, πr^2 means π (pi) × radius × radius.

area of a circle — π is a fixed value — radius

$$\text{area} = \pi r^2$$

Substitute the known values into the formula, in this example the radius is 4cm.

$$\text{area} = 3.14 \times 4^2$$

π is 3.14 to 3 significant figures; a more accurate value can be found on a calculator

this means 4×4

Multiply the radius by itself as shown – this makes the last multiplication simpler.

$$\text{area} = 3.14 \times 16$$

$4 \times 4 = 16$

Make sure the answer is in the right units (cm^2 here) and round it to a suitable number.

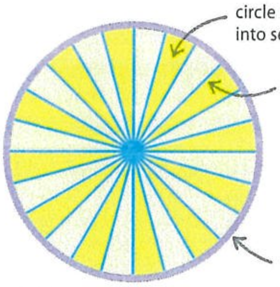
$$\text{area} = 50.24\text{cm}^2$$

answer is exactly 50.24

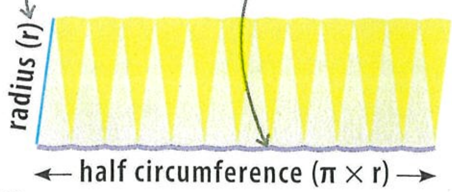
LOOKING CLOSER

Why does the formula for the area of a circle work?

The formula for the area of a circle can be proved by dividing a circle into segments, and rearranging the segments into a rectangular shape. The formula for the area of a rectangle (height × width) is simpler than that of the area for a circle. The rectangular shape's height is simply the length of a circle segment, which is the same as the radius of the circle. The width of the rectangular shape is half of the total segments, equivalent to half the circumference of the circle.



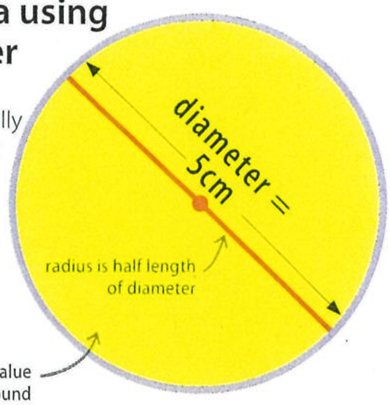
Split any circle up into equal segments, making them as small as possible.



Lay the segments out in a rectangular shape. The area of a rectangle is height × width, which in this case is radius × half circumference, or $\pi r \times r$, which is πr^2.

Finding area using the diameter

The formula for the area of a circle usually uses the radius, but the area can also be found if the diameter is given.



the area is the value that needs to be found

The formula for the area of a circle is always the same, whatever values are known.

$$\text{area} = \pi r^2$$

Substitute the known values into the formula – the radius is 2.5 in this example; half the diameter.

$$\text{area} = 3.14 \times 2.5^2$$

the radius is half the diameter: $5 \div 2 = 2.5$

Multiply the radius by itself (square it) as shown by the formula – this makes the last multiplication simpler.

π is 3.14 to 3 significant figures

$$\text{area} = 3.14 \times 6.25$$

$2.5 \times 2.5 = 6.25$

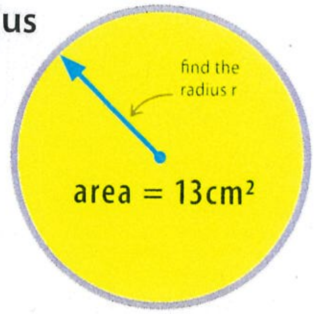
Make sure the answer is in the right units, cm^2 in this example, and round it to a suitable number.

19.6349... is rounded to 2 decimal places

$$\text{area} = 19.63\text{cm}^2$$

Finding the radius from the area

The formula for area of a circle can also be used to find the radius of a circle if its area is given.



The formula for the area of a circle can be used to find the radius if the area is known.

$$\text{area} = \pi r^2$$

Substitute the known values into the formula – here the area is 13cm.

$$13 = 3.14 \times r^2$$

Rearrange the formula so r^2 is on its own on one side – divide both sides by 3.14.

divide this side by 3.14

$$\frac{13}{3.14} = r^2$$

r^2 was multiplied by 3.14, so divide by 3.14 to isolate r^2

Round the answer, and switch the sides so that the unknown, r^2, is shown first.

r^2 is shown first

$$r^2 = 4.14$$

4.1380... is rounded to 2 decimal places

Find the square root of the last answer in order to find the value of the radius.

$$r = \sqrt{4.14}$$

Make sure the answer is in the right units (cm^2 here) and round it to a suitable number.

2.0342... is rounded to 2 decimal places

$$= 2.03\text{cm}$$

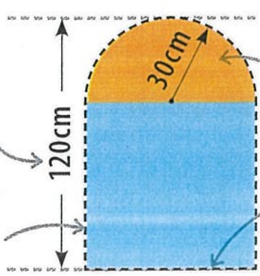
LOOKING CLOSER

More complex shapes

When two or more different shapes are put together, the result is called a compound shape. The area of a compound shape can be found by adding the areas of the parts of the shape. In this example, the two different parts are a semicircle, and a rectangle. The total area is 1,414cm² (area of the semicircle, which is $\frac{1}{2} \times \pi r^2$, half the area of a circle) + 5,400cm² (the area of the rectangle) = 6,814cm².

the total height of the shape

the height of the rectangle is $120 - 30 = 90$cm



the radius of the semicircle

the width of the rectangle is the same as the diameter of the circle. This can be be found by multiplying the radius by 2, $30 \times 2 = 60$cm

◁ **Compound shapes**
This compound shape consists of a semicircle and a rectangle. Its area can be found using only the two measurements given here.

Reference section

Mathematical signs and symbols

This table shows a selection of signs and symbols commonly used in mathematics. Using signs and symbols, mathematicians can express complex equations and formulas in a standardized way that is universally understood.

Symbol	Definition
+	plus; positive
−	minus; negative
±	plus or minus; positive or negative; degree of accuracy
∓	minus or plus; negative or positive
×	multiplied by (6 × 4)
·	multiplied by (6·4); scalar product of two vectors (A·B)
÷	divided by (6 ÷ 4)
/	divided by; ratio of (6/4)
—	divided by; ratio of ($\frac{6}{4}$)
○	circle
▲	triangle
□	square
▭	rectangle
▱	parallelogram
=	equals
≠, ≠	not equal to
≡	identical with; congruent to
≢, ≢	not identical with
≙	corresponds to

Symbol	Definition
:	ratio of (6:4)
::	proportionately equal (1:2::2:4)
≈, ≑, ≏	approximately equal to; equivalent to; similar to
≅	congruent to; identical with
>	greater than
≫	much greater than
≯	not greater than
<	less than
≪	much less than
≮	not less than
⩾, ≧, ⋝	equal to or greater than
⩽, ≦, ⋜	equal to or less than
∝	directly proportional to
( )	parentheses, can mean multiply
—	vinculum: division (a-b); chord of circle or length of line (AB);
$\overrightarrow{AB}$	vector
$\overline{AB}$	line segment
$\overleftrightarrow{AB}$	line

Symbol	Definition
∞	infinity
n^2	squared number
n^3	cubed number
n^4, n^5, etc	power, index
$\sqrt[2]{\ }$	square root
$\sqrt[3]{\ }$, $\sqrt[4]{\ }$	cube root, fourth root, etc.
%	per cent
°	degrees (°C); degree of arc, for example 90°
$\angle$, $\angle^s$	angle(s)
≚	equiangular
π	(pi) the ratio of the circumference to the diameter of a circle = 3.14
α	alpha (unknown angle)
θ	theta (unknown angle)
⊥	perpendicular
∟	right angle
$\parallel$, ⋕	parallel
∴	therefore
∵	because
$\stackrel{m}{=}$	measured by

Prime numbers

A prime number is any number that can only be exactly divided by 1 and itself without leaving a remainder. By definition, 1 is not a prime. There is no one formula for yielding every prime. Shown here are the first 250 prime numbers.

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199	211	223	227	229
233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349
353	359	367	373	379	383	389	397	401	409
419	421	431	433	439	443	449	457	461	463
467	479	487	491	499	503	509	521	523	541
547	557	563	569	571	577	587	593	599	601
607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733
739	743	751	757	761	769	773	787	797	809
811	821	823	827	829	839	853	857	859	863
877	881	883	887	907	911	919	929	937	941
947	953	967	971	977	983	991	997	1009	1013
1019	1021	1031	1033	1039	1049	1051	1061	1063	1069
1087	1091	1093	1097	1103	1109	1117	1123	1129	1151
1153	1163	1171	1181	1187	1193	1201	1213	1217	1223
1229	1231	1237	1249	1259	1277	1279	1283	1289	1291
1297	1301	1303	1307	1319	1321	1327	1361	1367	1373
1381	1399	1409	1423	1427	1429	1433	1439	1447	1451
1453	1459	1471	1481	1483	1487	1489	1493	1499	1511
1523	1531	1543	1549	1553	1559	1567	1571	1579	1583

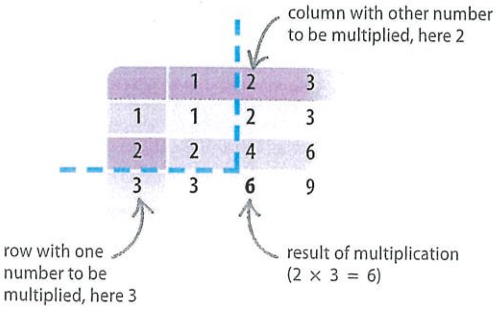
Squares, cubes, and roots

The table below shows the square, cube, square root, and cube root of whole numbers, to 3 decimal places.

No.	Square	Cube	Square root	Cube root
1	1	1	1.000	1.000
2	4	8	1.414	1.260
3	9	27	1.732	1.442
4	16	64	2.000	1.587
5	25	125	2.236	1.710
6	36	216	2.449	1.817
7	49	343	2.646	1.913
8	64	512	2.828	2.000
9	81	729	3.000	2.080
10	100	1,000	3.162	2.154
11	121	1,331	3.317	2.224
12	144	1,728	3.464	2.289
13	169	2,197	3.606	2.351
14	196	2,744	3.742	2.410
15	225	3,375	3.873	2.466
16	256	4,096	4.000	2.520
17	289	4,913	4.123	2.571
18	324	5,832	4.243	2.621
19	361	6,859	4.359	2.668
20	400	8,000	4.472	2.714
25	625	15,625	5.000	2.924
30	900	27,000	5.477	3.107
50	2,500	125,000	7.071	3.684

Multiplication table

This multiplication table shows the products of each whole number from 1 to 12, multiplied by each whole number from 1 to 12.



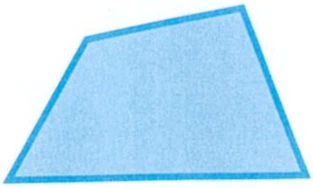
	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Shapes

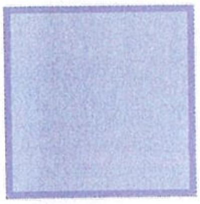
Two-dimensional shapes with straight lines are called polygons. They are named according to the number of sides they have. The number of sides is also equal to the number of interior angles. A circle has no straight lines, so it is not a polygon, although it is a two-dimensional shape.

△ **Circle**
A shape formed by a curved line that is always the same distance from a central point.

△ **Triangle**
A polygon with three sides and three interior angles.



△ **Quadrilateral**
A polygon with four sides and four interior angles.



△ **Square**
A quadrilateral with four equal sides and four equal interior angles of 90° (right angles).

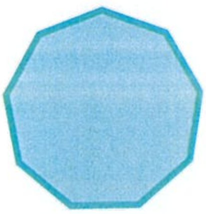
△ **Rectangle**
A quadrilateral with four equal interior angles and opposite sides of equal length.

△ **Parallelogram**
A quadrilateral with two pairs of parallel sides and opposite sides of equal length.

△ **Pentagon**
A polygon with five sides and five interior angles.

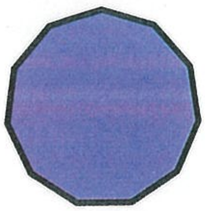
△ **Hexagon**
A polygon with six sides and six interior angles.

△ **Heptagon**
A polygon with seven sides and seven interior angles.



△ **Nonagon**
A polygon with nine sides and nine interior angles.

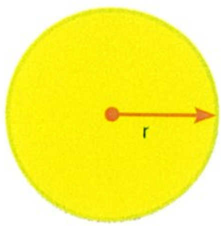
△ **Decagon**
A polygon with ten sides and ten interior angles.



△ **Hendecagon**
A polygon with eleven sides and eleven interior angles.

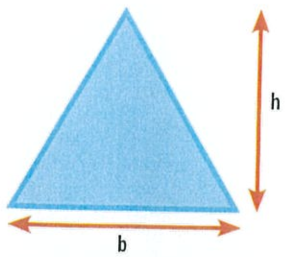
Area

The area of a shape is the amount of space inside it. Formulas for working out the areas of common shapes are given below.



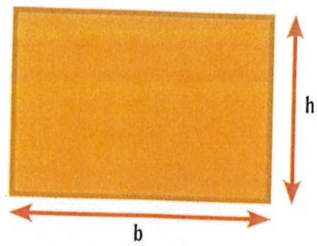
$$\text{area} = \pi r^2$$

△ **Circle**
The area of a circle equals pi ($\pi = 3.14$) multiplied by the square of its radius.



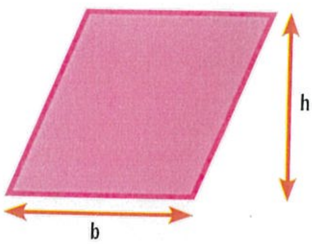
$$\text{area} = \frac{1}{2}bh$$

△ **Triangle**
The area of a triangle equals half multiplied by its base multiplied by its vertical height.



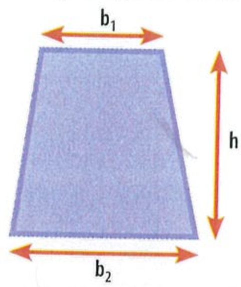
$$\text{area} = bh$$

△ **Rectangle**
The area of a rectangle equals its base multiplied by its height.



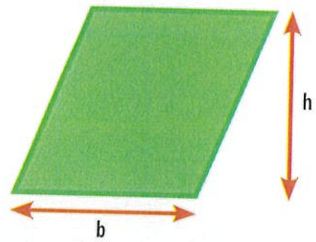
$$\text{area} = bh$$

△ **Parallelogram**
The area of a parallelogram equals its base multiplied by its vertical height.



$$\text{area} = \frac{1}{2}h(b_1+b_2)$$

△ **Trapezium**
The area of a trapezium equals the sum of the two parallel sides, multiplied by the vertical height, then multiplied by $^1/_2$.



$$\text{area} = bh$$

△ **Rhombus**
The area of a rhombus equals its base multiplied by its vertical height.

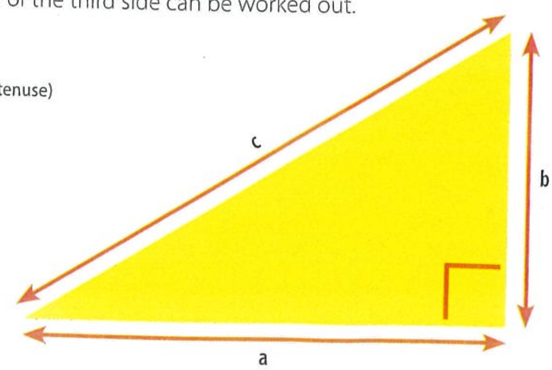
Pythagoras' theorem

This theorem relates the lengths of all the sides of a right-angled triangle, so that if any two sides are known, the length of the third side can be worked out.

side a　　　side c (hypotenuse)

$$a^2 + b^2 = c^2$$

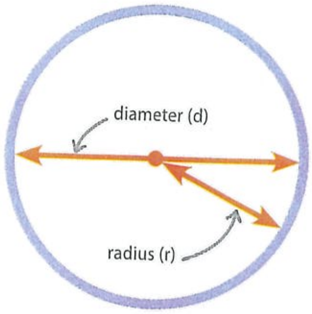
side b



◁ **The theorem**
In a right-angled triangle the square of the hypotenuse (the largest side, c) is the sum of the squares of the other two sides (a and b).

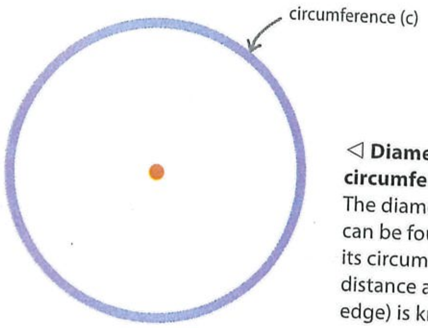
Parts of a circle

Various properties of a circle can be measured using certain characteristics, such as the radius, circumference, or length of an arc, with the formulas given below. Pi (π) is the ratio of the circumference to the diameter of a circle; pi is equal to 3.14 (to 2 decimal places).



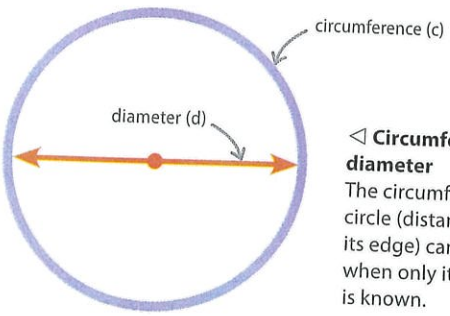
◁ **Diameter and radius**
The diameter of a circle is a straight line running right across the circle and through its centre. It is twice the length of the radius (the line from the centre to the circumference).

$$\text{diameter} = 2r$$



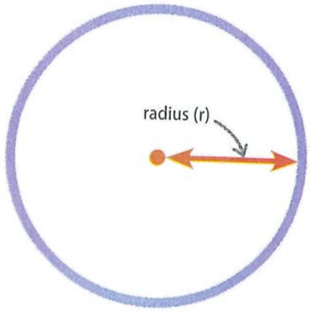
◁ **Diameter and circumference**
The diameter of a circle can be found when only its circumference (the distance around the edge) is known.

$$\text{diameter} = \frac{c}{\pi}$$



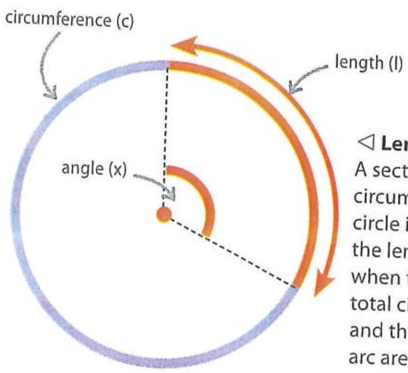
◁ **Circumference and diameter**
The circumference of a circle (distance around its edge) can be found when only its diameter is known.

$$\text{circumference} = \pi d$$



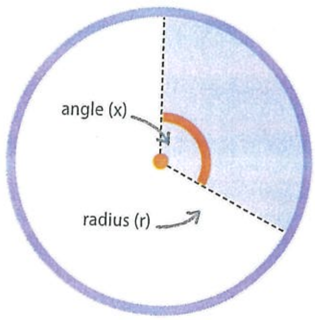
◁ **Circumference and radius**
The circumference of a circle (distance around its edge) can be found when only its radius is known.

$$\text{circumference} = 2\pi r$$



◁ **Length of an arc**
A section of the circumference of a circle is known as an arc, the length can be found when the circle's total circumference and the angle of the arc are known.

$$\text{length of an arc} = \frac{x}{360} \times c$$



◁ **Area of a sector**
The area of a sector (or "slice") of a circle can be found when the circle's area and the angle of the sector are known.

$$\text{area of a sector} = \frac{x}{360} \times \pi r^2$$