Stage 5

Measurement:

Area & Surface Area

Booklet 3a

Name:

Date Started:

Date Completed:

Acknowledgments

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Outcomes

By completing this unit, students are working towards achieving the following outcomes.

You have the opportunity to learn to:

- use appropriate terminology, diagrams and symbols in mathematical contexts **MA5.1-1WM**
- select and use appropriate strategies to solve problems **MA5.1-2WM**
- calculate the areas of composite shapes, and the surface areas of rectangular and triangular prisms **MA5.1-8MG**
- select appropriate notations and conventions to communicate mathematical ideas and solutions **MA5.2-1WM**
- interpret mathematical or real-life situations, systematically applying appropriate strategies to solve problems **MA5.2-2WM**
- calculate the surface areas of right prims, cylinders and related composite solids **MA5.2-11MG**
- apply formulas to calculate the volumes of composite solids composed of right prisms and cylinders **MA5.2-14MG**

Knowledge, skills and understanding

Students:

Working Mathematically
- develop understanding and fluency in mathematics through inquiry, exploring and connecting mathematical concepts, choose and apply problem-solving skills and mathematical techniques, communication and reasoning

Measurement and Geometry
- identify, visualise and quantify measures and the attributes of shapes and objects, and explore measurement concepts and geometric relationships, applying formulas, strategies and geometric reasoning in the solution of problems

Measurement: Area & Surface Area

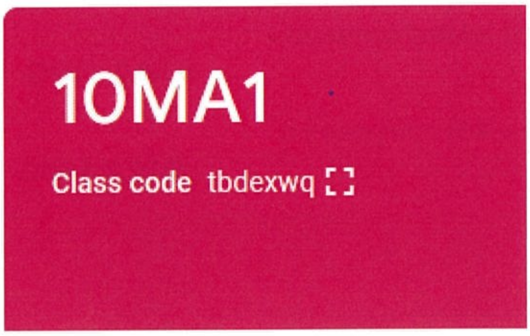
Hi Stage 5,

Hope you are all staying safe and healthy at home ☺

The booklet is a bit different (and longer!) this term because I want you to have an explanation to read before each exercise (just like the notes we would normally write up on the board). It is really important to read and understand the explanation pages before you go ahead and start the exercises (even if you already know how to do it).

If you are feeling confused or need some help either leave a comment on our Stage 5 google classroom (I know it says just 10MA1 but it is for both classes ☺) or email your question to me at my school email:

brittany.singleton@det.nsw.edu.au



Contents

Part A: Perimeter & Pythagoras Revision

In this booklet you will be solving problems involving perimeters of plane shapes including composite shapes.

The ability to determine the perimeters of two-dimensional shapes is of fundamental importance in many everyday situations, such as framing a picture, furnishing a room, fencing a garden or a yard, and measuring land for farming or building construction.

Indicators

By the end of this booklet, you will have been given the opportunity to work towards aspects of knowledge and skills including:

- finding the perimeters of a range of plane shapes, including parallelograms, trapeziums, rhombuses, kites and simple composite figures
- comparing perimeters of rectangles with the same area
- solving problems involving the perimeters of plane shapes

Use any blank space in the booklet to show your working out or write down anything you are struggling with!

Perimeter

A **plane shape** is any flat shape.

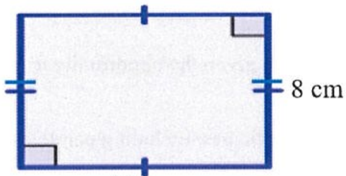
The **perimeter** of a plane shape is the sum of the lengths of the sides. In other words, it is the distance around the edge of the shape.

You can find the perimeter of a shape by either adding up the lengths around the outside of the shape or by recognising that because there are some equal sides in your shape, you can group these equal sides together.

Follow through the steps in this example.

Calculate the perimeter of the following shapes (these shapes are not drawn to scale):

a)



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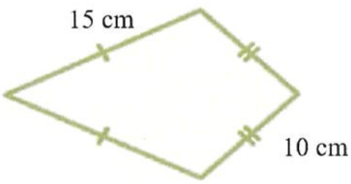


Solution

a) This is a rectangle, with equal opposite sides. We can calculate the perimeter by either adding each side or multiplying lots of equal sides

$$\text{Perimeter} = 10 + 8 + 10 + 8 = 36 \text{ cm}$$

or

$$\text{Perimeter} = 2 \times 8 + 2 \times 10 = 36 \text{ cm}$$

b) A kite has adjacent equal sides.

$$\text{Perimeter} = 2 \times 15 + 2 \times 10 = 50 \text{ cm}$$

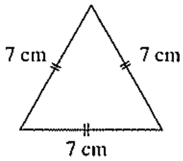
or

$$\text{Perimeter} = 15 + 10 + 15 + 10 = 50 \text{ cm}$$

Exercise A.1

Question 1 Find the perimeter of each of the following triangles.

a



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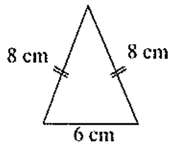

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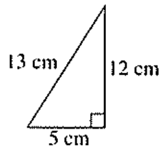

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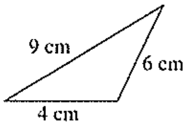

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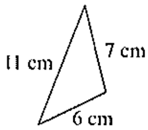

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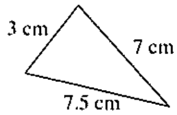
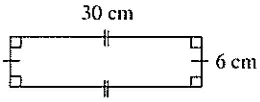


Question 2 Find the perimeter of the following squares and rectangles.

a



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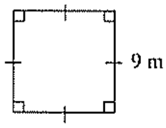

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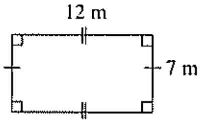

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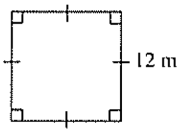

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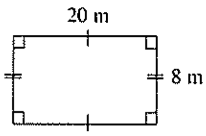

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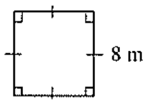
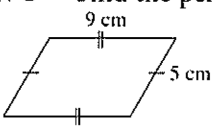


Question 3 Find the perimeter of the following.

a



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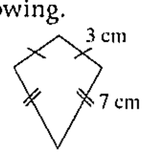

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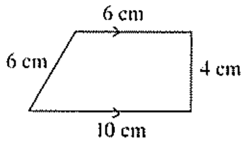

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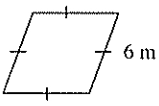

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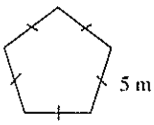

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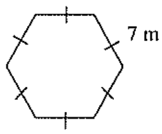

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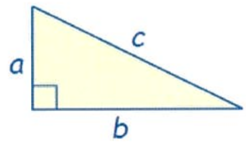
How Do I Use it?

Write it down as an equation:



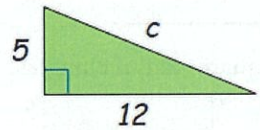
$a^2 + b^2 = c^2$

Remember!!!

'c' is always the longest slopey side.

Then we use algebra to find any missing value, as in these examples:

Example: Solve this triangle



Start with: $a^2 + b^2 = c^2$

Put in what we know: $5^2 + 12^2 = c^2$

Calculate squares: $25 + 144 = c^2$

25+144=169: $169 = c^2$

Swap sides: $c^2 = 169$

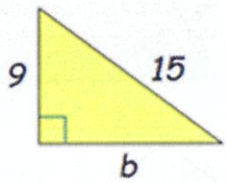
Square root of both sides: $c = \sqrt{169}$

Calculate: $\mathbf{c = 13}$

We can find one of the shorter sides using:

$a^2 = c^2 - b^2$

Example: Solve this triangle.



$a^2 = c^2 - b^2$

$b^2 = 15^2 - 9^2$

$= 225 - 81$

$= 144$

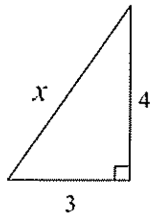
$b = \sqrt{144}$

$= 12$

Exercise A.2

QUESTION 1 Find the length of the hypotenuse in each of the following. (All measurements are in centimetres.)

a



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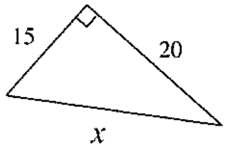

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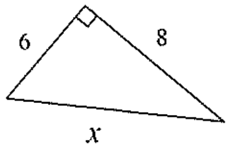

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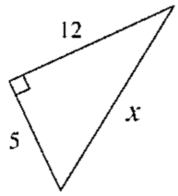

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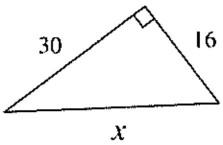

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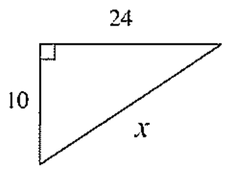
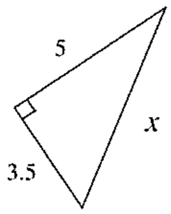


QUESTION 2 Find the length of the hypotenuse correct to one decimal place. (All measurements are in centimetres.)

a



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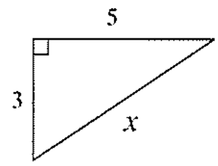

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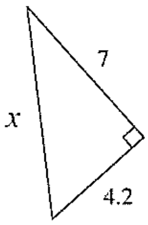

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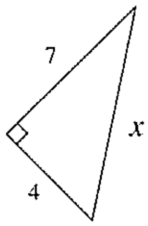

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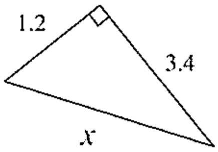

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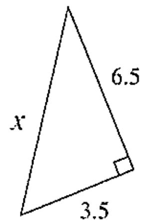
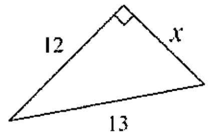


Exercise A.3

QUESTION 1 In the following triangles find the length of the unknown sides. (All measurements are in centimetres.)

a



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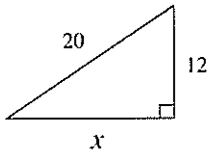

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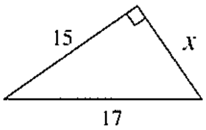

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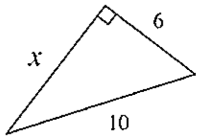

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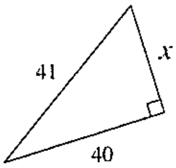

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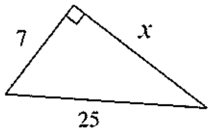
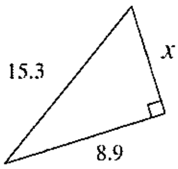


QUESTION 2 Find the length of the unknown side correct to one decimal place. (All measurements are in centimetres.)

a



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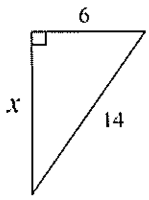

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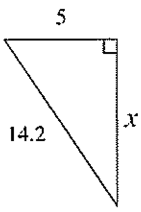

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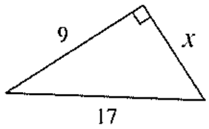

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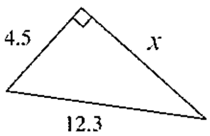

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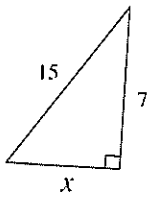
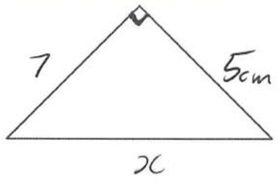


Pythagoras and Perimeter

Pythagoras lets us find the lengths of missing sides in right angle triangles, we can then use these lengths to calculate the perimeter of different shapes by adding up the side lengths. See the example to the right:



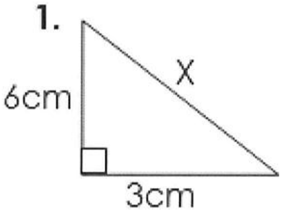
$c^2 = a^2 + b^2$
$x^2 = 7^2 + 5^2$
$x^2 = 49 + 25$
$x^2 = 74$
$x = \sqrt{74}$
$= 8.6$ (1dp)

$\therefore P = 7 + 5 + 8.6$
$= 20.6$ cm

Exercise A.4

Find the length of the missing side (round answers to 1dp if necessary) and then calculate the perimeter of each triangle.

1.



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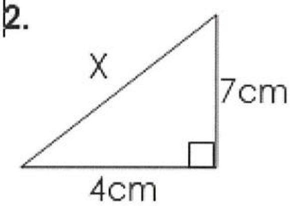

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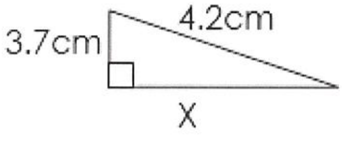

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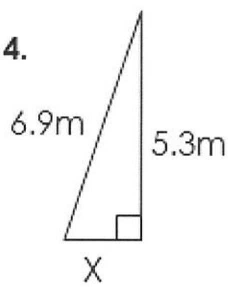
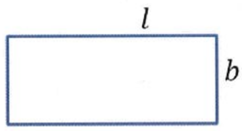
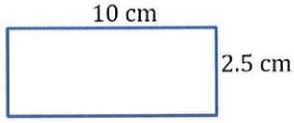
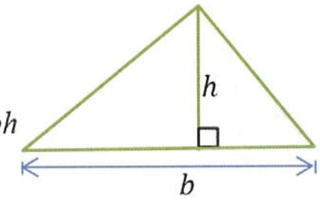
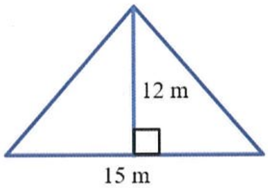
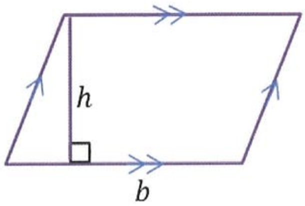
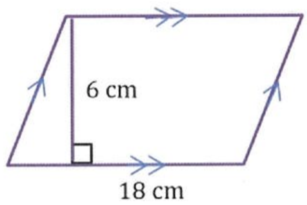
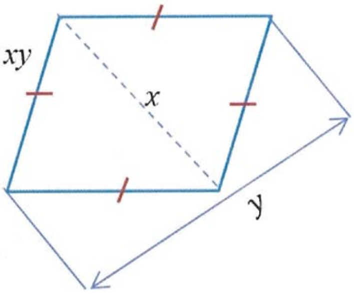
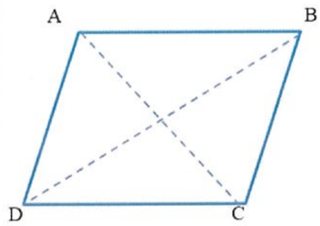
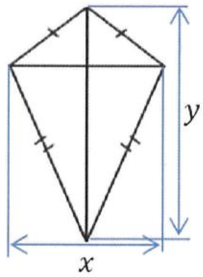
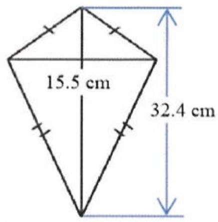
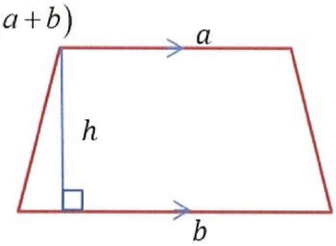
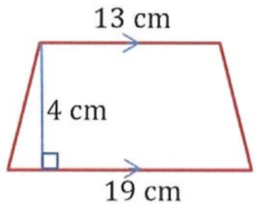


Part B: Area

Revision of Area Formulas

Formula	Examples
Rectangle $area = length \times breadth$ $A = lb$ 	 $A = lb$ $= 10 \times 2.5$ $= 25\ cm^2$
Triangle $area = \frac{1}{2} \times base \times height$ $A = \frac{1}{2} \times bh$ 	 $A = \frac{1}{2} \times bh$ $= \frac{1}{2} \times 15 \times 12$ $= 90\ m^2$
Parallelogram $area = base \times height$ $A = bh$ 	 $A = bh$ $= 18 \times 6$ $= 108\ m^2$
Rhombus $area = \frac{1}{2} \times diagonal \times\ other\ diagonal$ $A = \frac{1}{2}xy$ 	 $AC = 16\ cm$ $BD = 22\ cm$ $A = \frac{1}{2}xy$ $= \frac{1}{2} \times 22 \times 16$ $= 176\ cm^2$

Formula	Examples
Kite $area = \frac{1}{2} \times diagonal \times diagonal$ $A = \frac{1}{2}xy$ 	 $A = \frac{1}{2} \times xy$ $= \frac{1}{2} \times 15.5 \times 32.4$ $= 251.1\ cm^2$
Trapezium $area = \frac{1}{2} \times height \times sum\ of\ parallel\ sides$ $A = \frac{1}{2}h(a+b)$ 	 $A = \frac{1}{2}h(a+b)$ $= \frac{1}{2} \times 4(19+13)$ $= 2 \times 32$ $= 64\ cm^2$

Exercise B.1

1. Use the formula page to calculate the area of:

   a) a square that has a side length of 7 cm.

   ____

   ____

   b) a rectangle that has a length of 9 mm and a breadth of 4 mm.

   ____

   ____

   c) a triangle that has a perpendicular height of 15 mm and a base length of 8 mm.

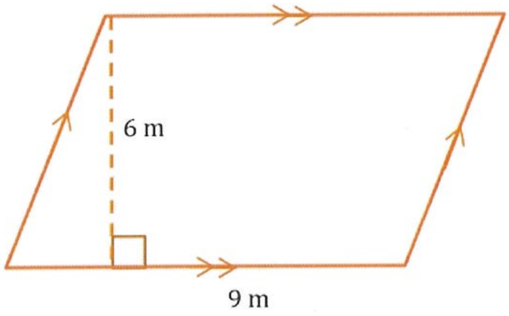
   ____

   ____

   ____

2. Calculate the area of the following figures. Identify the shape first then look for the formula on the previous pages.

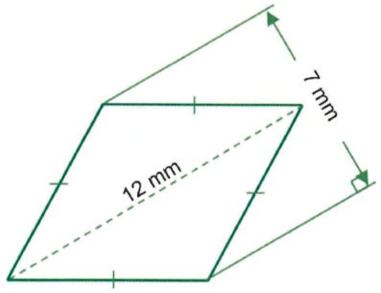
   a)



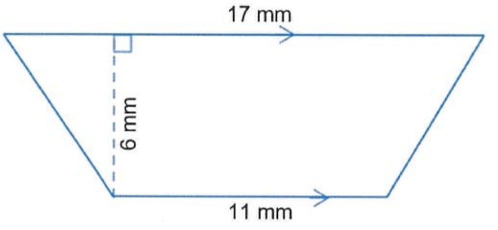
   ____

   ____

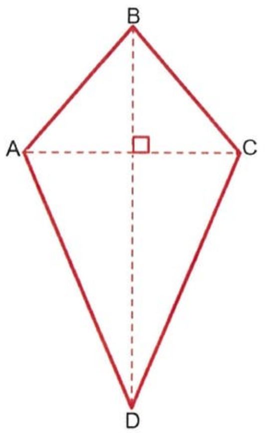
b)



c)



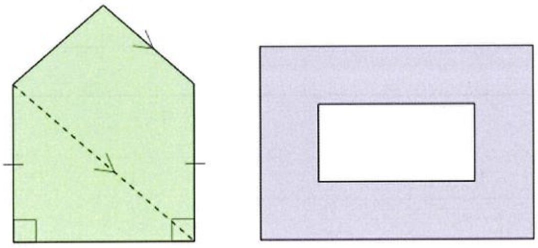
d)



AC = 10 m
BD = 18 m

Area of composite figures

Composite figures can have more than one shape inside them. They also may have shapes cut out of them. Here are two examples.

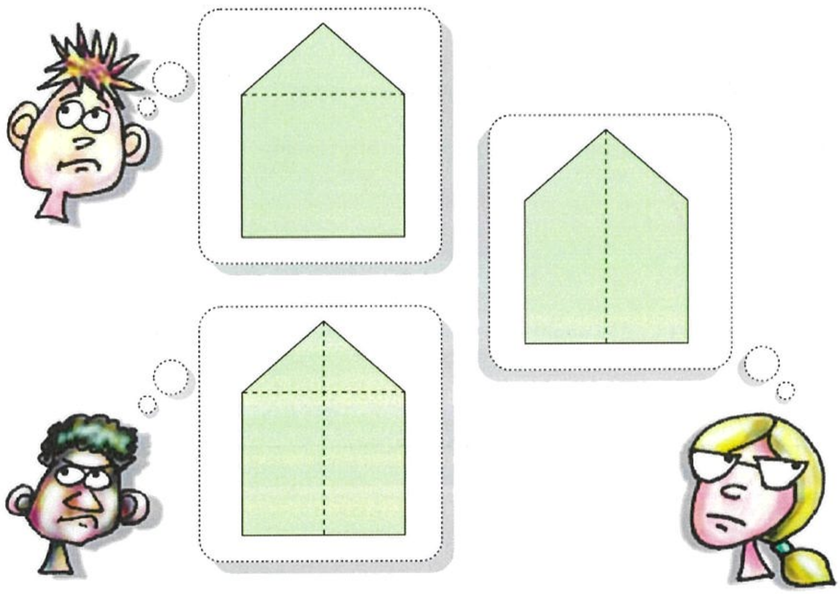


The composite areas are shaded.

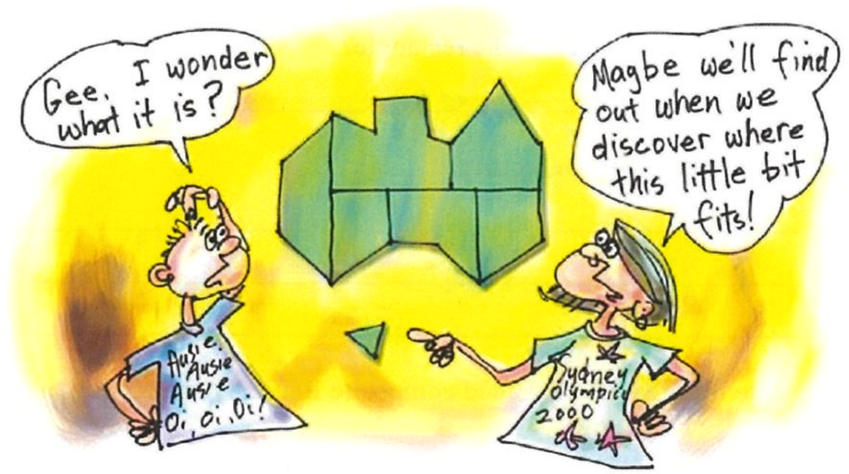
To calculate the area of the first figure, you must add the triangular area and trapezium area together. To calculate the area of the second figure you must subtract the smaller rectangular area from the larger rectangular area.

Some figures can be divided different ways.

Can you think of the first figure above in a different way?



There are many different shapes in composite figures.

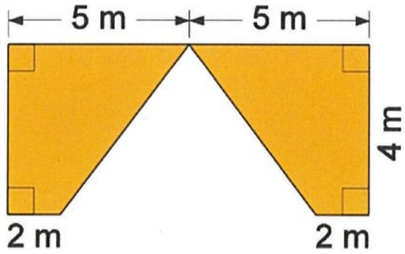


However, when calculating the area it is always best to think carefully about what shapes you are going to choose. Choosing the smallest number of shapes means you have less calculations to do.

Activity – Composite figures

Try these.

1. Answer the following questions about the figure below.



The rectangle above has a triangle cut out of it.

Draw a dotted line along the base of the triangle so you can see it more clearly.

a) How long are the base and height of this triangle?

____

____

Indicate exactly where these dimensions are, on the diagram above.

b) Calculate the area of the triangle.

____

____

____

c) What are the dimensions of the rectangle?

____

d) Calculate the area of the rectangle.

____

____

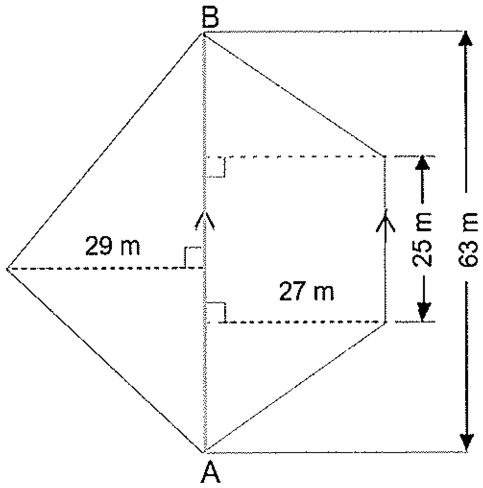
____

e) Calculate the area of the shaded composite figure by subtracting one area from the other.

____

____

2. A property was surveyed along its length from corner A to corner B. The diagram below shows the results of the survey.



(This is called a traverse survey.)

a) There are many shapes inside this figure. However, there are two that make up the whole of this property. Write the names of the two shapes.

____

b) Calculate the area of the property by finding the area of these two shapes first, then adding these areas together.

____

____

____

____

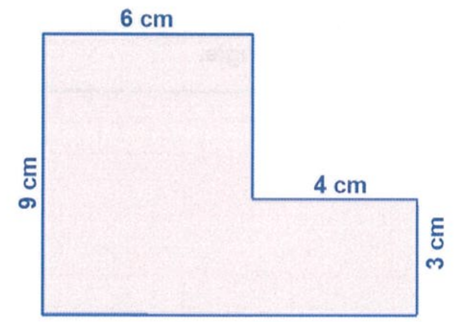
____

____

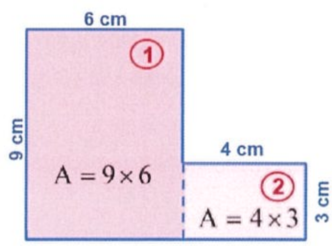
Further composite area and practical problems

Some composite shapes can be divided several ways. It does not make any difference how you divide the shape as long as you can correctly find the area of each shape.

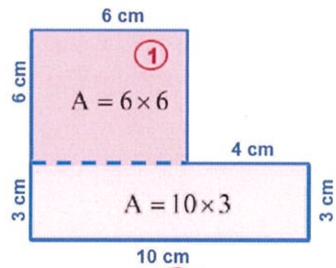
Consider this figure.



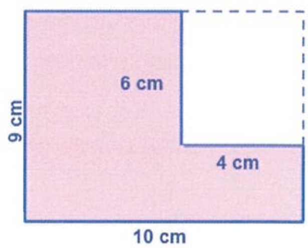
The composite shape above has been divided three different ways. In each case we have two rectangles. You might need to find some side lengths that are not given.



Ⓐ



Ⓑ



Ⓒ

We will check that the area is the same in Ⓐ, Ⓑ and Ⓒ

Ⓐ

$$
\begin{aligned}
\text{Area 1} &= 9\times 6 \\
&= 54\ \text{cm}^2 \\
\text{Area 2} &= 4\times 3 \\
&= 12\ \text{cm}^2 \\
\text{Total area} &= 54+12 \\
&= 66\ \text{cm}^2
\end{aligned}
$$

Ⓑ

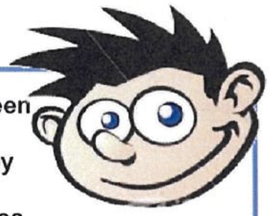
$$\begin{aligned}\text{Area } 1 &= 6\times 6\\ &= 36 \text{ cm}^2\\ \text{Area } 2 &= 10\times 3\\ &= 30 \text{ cm}^2\\ \text{Total Area} &= 36+30\\ &= 66 \text{ cm}^2\end{aligned}$$

Note: We had to do $6+4=10$ to find length of rectangle in Area 2

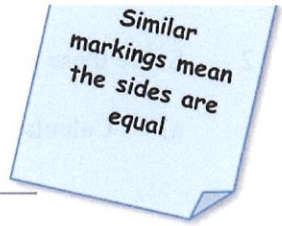
Ⓒ

$$\begin{aligned}\text{Area large rectangle} &= 10\times 9\\ &= 90 \text{ cm}^2\\ \text{Area of small rectangle} &= 6\times 4\\ &= 24 \text{ cm}^2\\ \text{Required area} &= 90-24\\ &= 66 \text{ cm}^2\end{aligned}$$

In this method, the shape has been completed to form a rectangle. The original area can be found by finding the area of the larger rectangle and subtracting the area of the small rectangle.

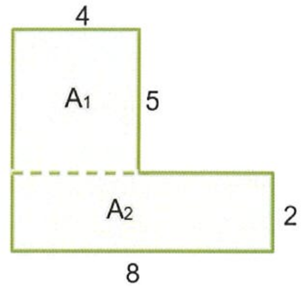


Exercise B.2



1. Find the area of each figure by dividing the shape. You may need to find some missing dimensions. All measurements are in metres.

a)



A_1 = ______________________

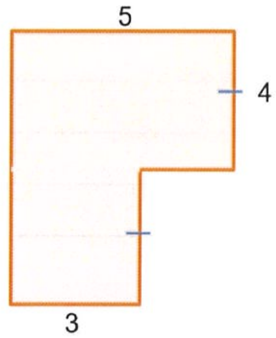
____

A_2 = ______________________

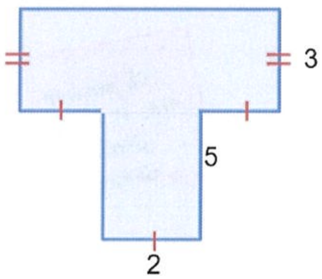
____

Total area = ______________________

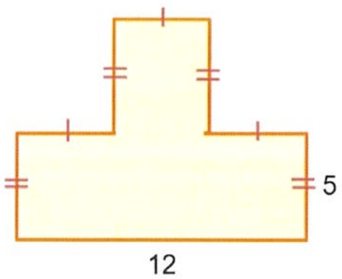
b)



c)



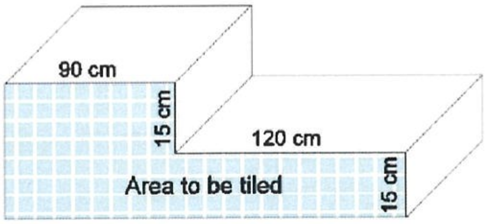
d)



Many practical situations such as landscaping, drawing house plans and home decorating require the area of more than one shape to be found.

2. Solve these problems by first drawing a diagram when needed.

a) Calculate the area of the side of this step that needs to be tiled.



____

____

____

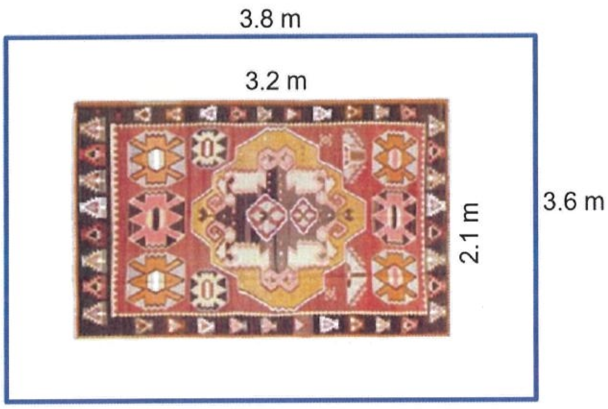
____

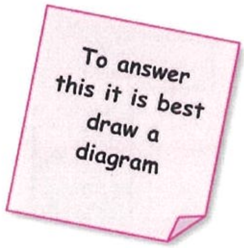
____

____

b) A rug measuring 3.2 metres by 2.1 metres is placed in a room measuring 3.8 metres by 3.6 metres.

i) What area of floor remains around the rug?





Area room = ____

____

Area rug = ______________________________

____

Remaining area = ______________________________

____

ii) What is the cost of the rug if it is made from carpet costing $62 per square metre?

____

____

____

c) A garden is made in the shape of a trapezium with parallel sides 2.4 m and 1.7 m, the distance between them being 3m.

i) Draw the trapezium and put on all given dimensions.

ii) Calculate the area of the garden.

____

____

____

iii) How many ferns can be planted in the garden if they each take up 0.3m^2 ?

____

____

____

iv) How much will the ferns cost if they are $13.50 each?

____

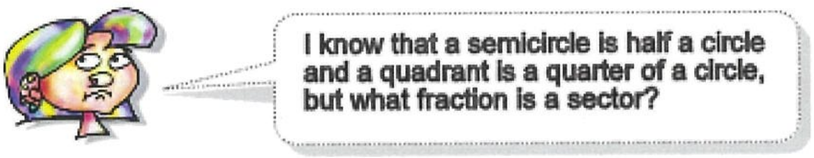
____

Area of a sector

A sector is any part of a circle that is bounded by two radii and an arc.

A sector looks just like a standard piece of pizza.

A semicircle and a quadrant are both special types of sectors.



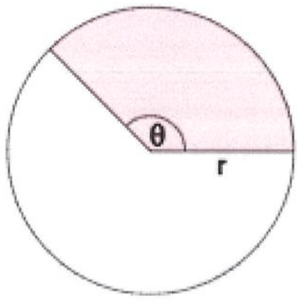
The class divide into groups to discuss this problem.

One group looks just at semicircles and quadrants and realises that the angle at the centre of a semicircle is $180°$. This is half of $360°$ because $\frac{180°}{360°} = \frac{1}{2}$. From this they realise that, for a quadrant, $\frac{90°}{360°} = \frac{1}{4}$. They conclude that it is impossible to calculate the area of a sector without knowing the angle between the two radii.

Another group use ratios to write these conclusions:

$$\text{Area of semicircle : area of circle} = 180 : 360$$
$$= 1 : 2$$
$$\text{Area of quadrant : area of circle} = 90 : 360$$
$$= 1 : 4$$

So if the angle between the two radii is θ°,



then, area of sector : area of circle = $\theta^\circ : 360^\circ$

Both ways of thinking are correct, so because the area of a whole circle is πr^2 , then

$$\text{area of semicircle} = \frac{180}{360}\pi r^2$$

$$\text{area of quadrant} = \frac{90}{360}\pi r^2$$

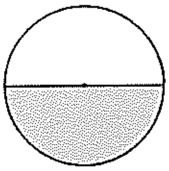
and

$$\text{area of sector} = \frac{\theta^\circ}{360}\pi r^2$$

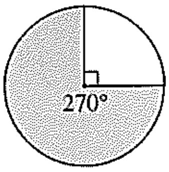
The activity on the next page helps you to understand this formula further.

QUESTION 1 These six circles have the same radius. List the black sectors in ascending order of area.

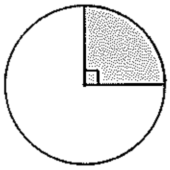
A



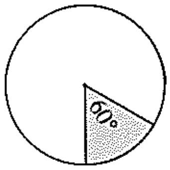
B



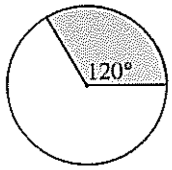
C



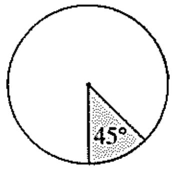
D



E

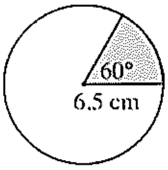


F

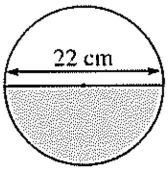


QUESTION 2 Calculate the area of each shaded sector correct to two significant figures.

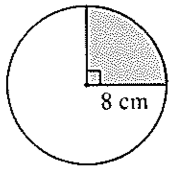
a



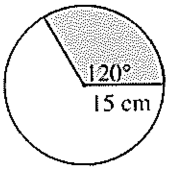
b



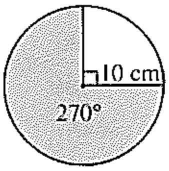
c



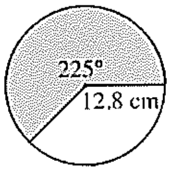
d



e

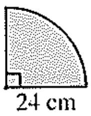


f

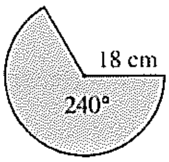


QUESTION 3 Find the area of each sector, leaving your answer in terms of π.

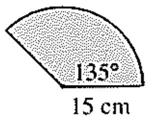
a



b

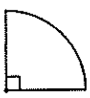


c



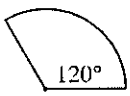
QUESTION 4 Find the radius, correct to one decimal place, for each sector.

a



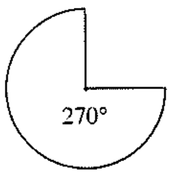
$A = 42\text{ cm}^2$

b



$A = 75\text{ cm}^2$

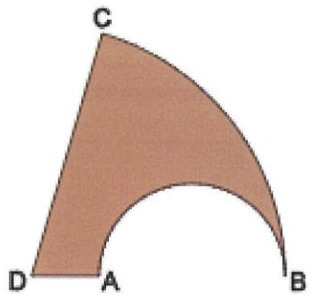
c



$A = 120\text{ cm}^2$

Sectors in composite figures

Composite figures can also contain sectors. Can you see where the sector is in the diagram below? How would you calculate the shaded area?



AB is the diameter of the small semicircle. D is the centre of another circle from which CB has been cut.

The sector is between DB, DC and the arc that joins C and B. To calculate the composite area you must subtract the area of the small semicircle from the area of the sector.

Try calculating this shaded area in the following activity.

Activity – Sectors in composite figures

Try this.

1. Calculate the area of the shaded figure on the previous page if $DC = 6\ cm$, $AB = 5\ cm$ and $\angle CDB = 80°$.

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You can calculate each area separately and then add or subtract these areas by using your calculator memory. (Reusing approximated answers can often lead to inaccurate answers.)

Alternatively, you can do all your calculations in one step by first writing a word equation using the names of shapes and then substituting all values in at the same time.

In the question in the previous activity your word equation would look something like this.

Shaded area = area of sector − area of semicircle

Now write the formulas for each of these areas, like this:

$$A = \frac{\theta°}{360}\pi R^2 - \frac{1}{2}\pi r^2,$$

(Note that the two radii are different so they should be given different letters.)

After this substitute in the measurements you know.

$$A = \frac{80}{360} \times \pi \times 6^2 - \frac{1}{2} \times \pi \times 2.5^2$$

Then use your calculator.

This is one way you can use your calculator:

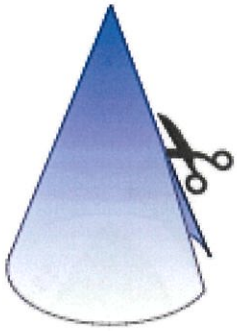
80 [÷] 360 [×] [π] [×] 6 [x^2] [−] 0.5 [×] [π] [×] 2.5 [x^2] [=]

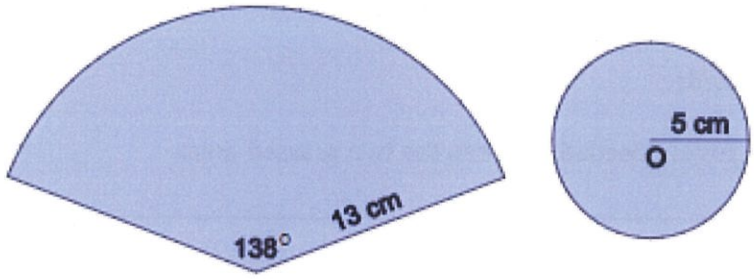
(Your answer is approximately equal to 15.3 cm^2.)

Write word equations in the following activity, then substitute as shown above so that you can do all your calculations at the end.

Ex B.3

1. A paper cone is cut around its base then up its side and placed out flat to form the sector and circle below.





O is the centre of the circular base.

Calculate the area of paper used in this cone.

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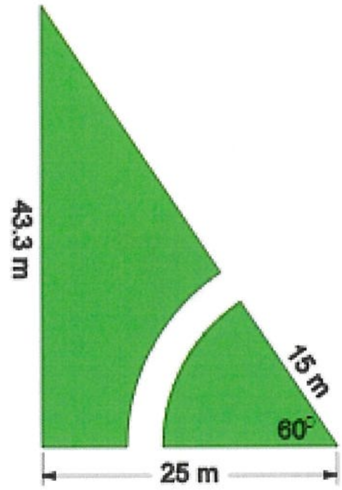
2. A plan of a proposed park is drawn below. It is to be constructed between three streets in the local area. (Two of these streets are at right angles.) It has a curved footpath through it that needs to be paved. One of the grassed areas is cut from a circle of radius 15 m.



The footpath is 3 m wide.

a) Calculate the paving needed between the two grassed areas.

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b) Calculate the area of grass, to the nearest square metre, that needs to be planted.

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c) If paving costs \$32 per square metre and turf cost \$7 per square metre, calculate the total cost of materials for this park.

____

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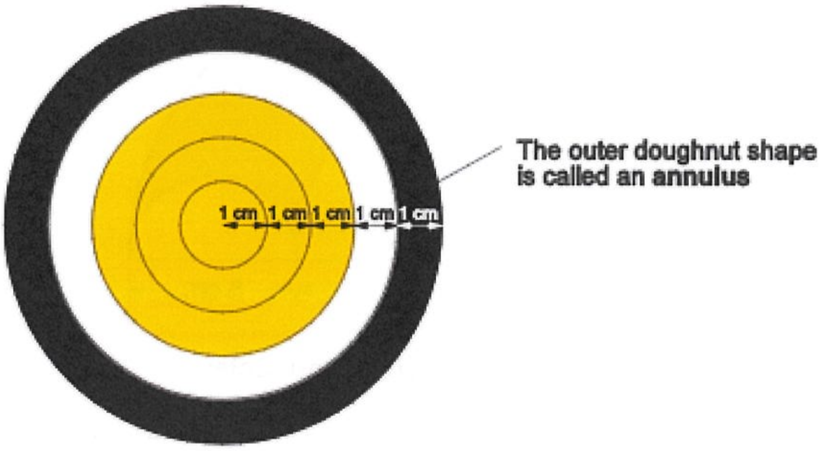
3. A circle with a radius of 1 cm has four more concentric circles drawn around it. The radius of each circle is 1 cm bigger than the radius of the circle inside it.



a) Which of the two areas indicated on the diagram (black or yellow) do you think is greater?

____

b) Calculate both these areas and compare them.

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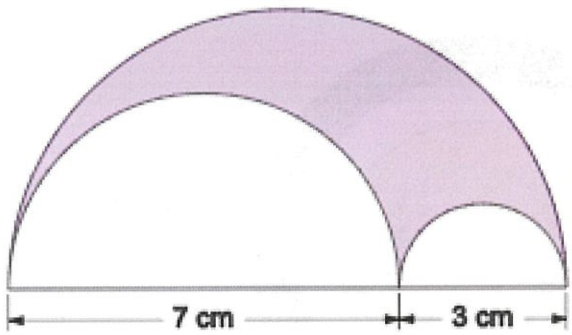
Surprised by the answer?

4. Use the diagram below to answer the following questions.



a) Calculate the total area of the two smaller semicircles.

b) Calculate the area of the shaded section of the figure above.

c) Compare this shaded area to the total of the two smaller semicircular areas.

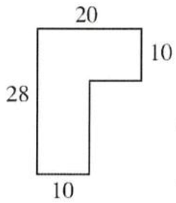
The shaded figure above is called an '**arbelos**'. This Greek word means 'shoemaker's knife' because it looks like the knives that ancient cobblers used to cut shoe leather.

Archimedes was the first mathematician to study the geometry of the arbelos.

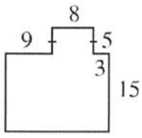
Ex B.4

Note: For some of these questions you may need some extra working out space do so underneath.

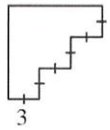
QUESTION 1 Find the area of each composite figure by dividing it into different shapes. All measurements are in cm and all angles are right angles.

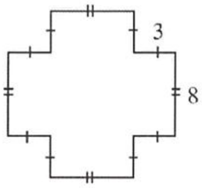


b

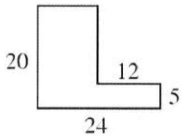


c

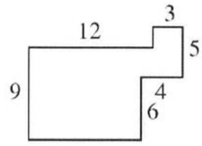




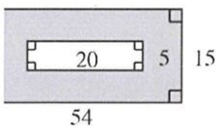
e



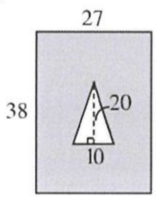
f



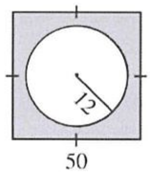
QUESTION 2 Find the area of the following shaded shapes. All measurements are in centimetres.

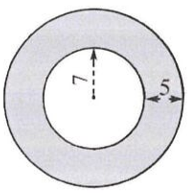


b

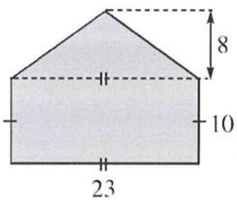


c

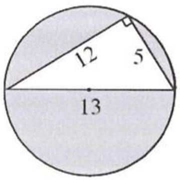




e

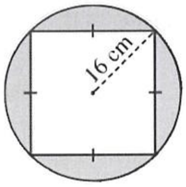


f

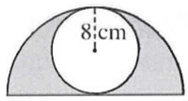


Note: For some of these questions you may need some extra working out space do so underneath.

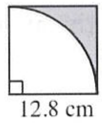
QUESTION 3 Find the area of each shaded shape.

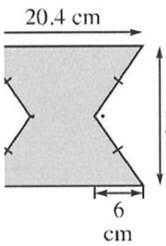


b

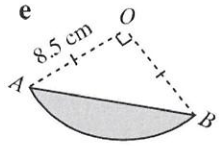


c



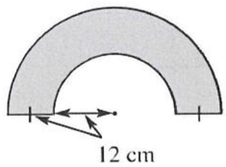


e



O is the centre of the circle with arc AB

f



Part C: Nets and Surface Area

Activity - Preliminary Quiz

Try these.

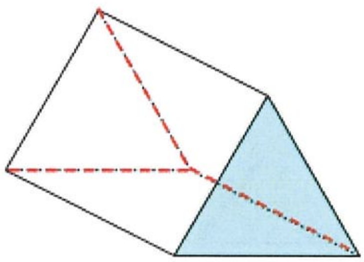
1. Name the following plane shapes.

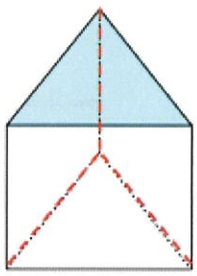
a) 

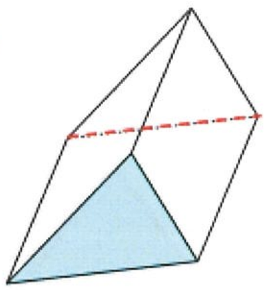
b) 

c) 

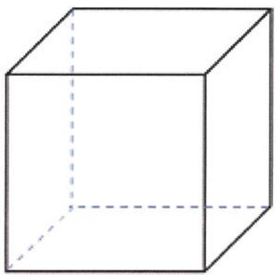
2. On the prism below, colour the face that has the same length and width as the face that is already shaded

a) 

b) 

c) 

3. Colour the back face of this cube.

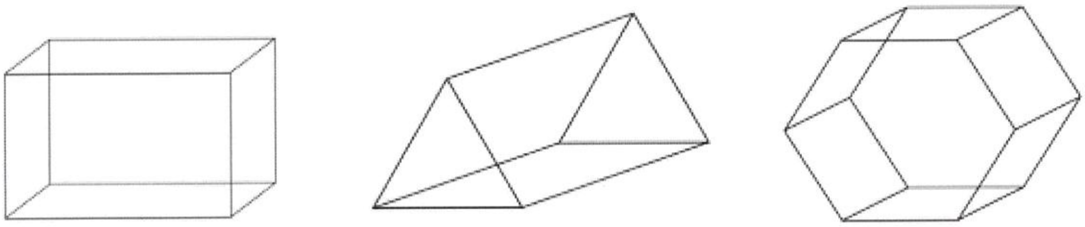


Naming and drawing prisms

A prism is a three-dimensional object that has a uniform cross-section in the shape of a polygon.

In this course we will only be considering right prisms which are prisms in which the base is parallel to the top face, and the sides meet the front face at a right angle.

Below are some examples of prisms.

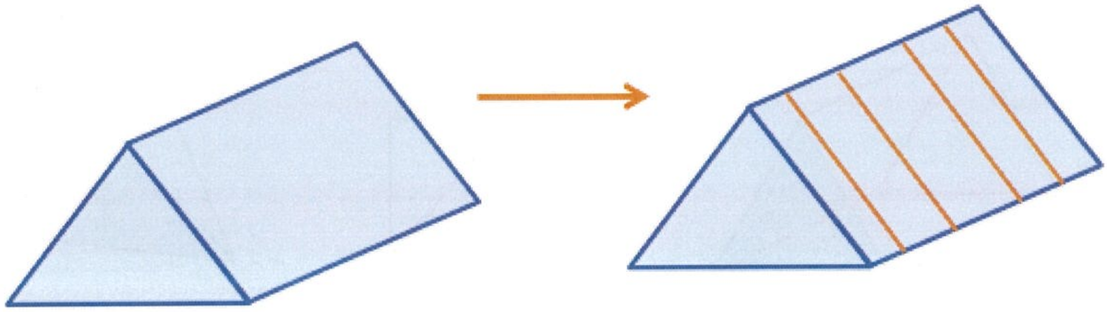


Rectangular prism **Triangular prism** **Hexagonal prism**

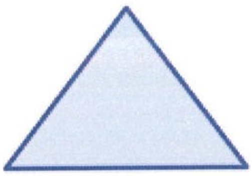
As you may have noticed from the examples above, prisms are named according to the shape of the cross-section.

The cross-section is the shape that remains unchanged if you were to cut the prism into slices along its length.

Consider the following prism:



If we were to cut it in slices, as shown in the diagram on the right, the cross section of each slice would be a triangle, as shown below.



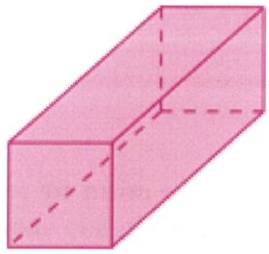
Hence this prism is a triangular prism.

Activity – Naming prisms

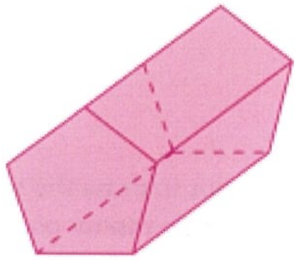
Try these.

1. Name each of the prisms shown below.

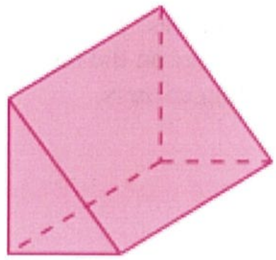
a)



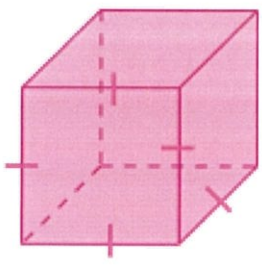
b)



c)



d)



Drawing prisms

Later in the course you will be looking at solving problems on finding the surface area of prisms. It is often helpful to be able to draw a diagram to go with the information you are given.

Follow the steps in the example below when drawing any prisms. Always use a ruler!

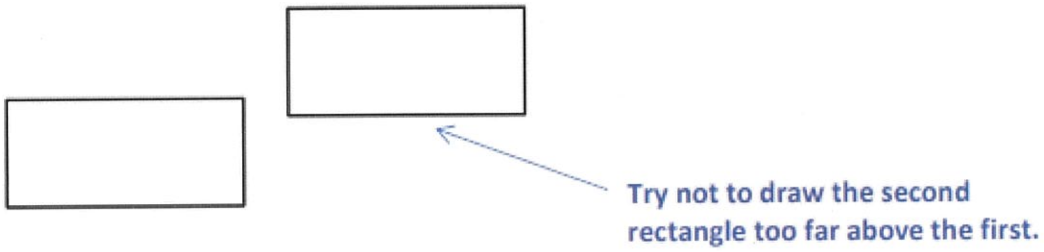
Example 1

Draw a rectangular prism.

Solution

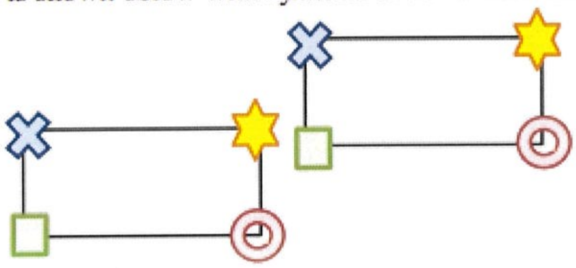
Step 1

A rectangular prism means that the 'front' and 'back' ends of the prism are rectangles.
Draw in the two rectangles first, with one sitting to the right, and slightly above the other.

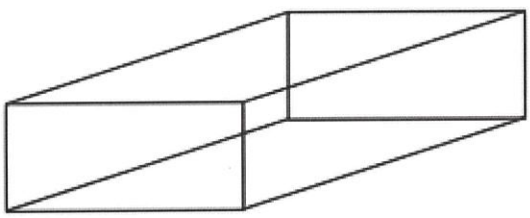


Step 2

Draw lines connecting the corresponding corners on the two faces. This means that the top left corner on the front rectangle will be joined to the top left corner on the back rectangle and so on. This is shown below with symbols to show the corresponding corners.



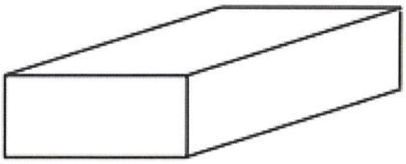
Connecting the corners:



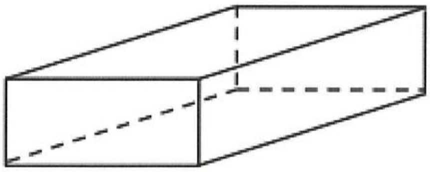
Your prism is now complete! The prism shown above shows all edges of the prism, even the edges that you would not see if you were looking at the solid shape in real life.

Depending on the prism, sometimes having all edges visible can make the solid harder to look at. There are two option on how to simplify your prism.

Option 1 – Only draw edges that would be seen if looking at the prism.



Option 2 – Draw edges that would usually be hidden with a dotted or dashed line.



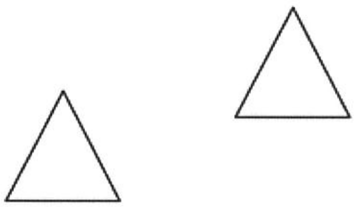
All three styles of drawing prisms are fine, and you may choose whichever is easiest for you. However, it will be important later in the course that you understand each form of diagram.

Activity – Drawing prisms

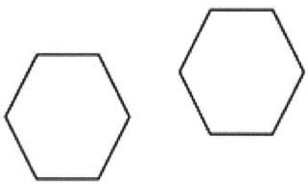
Try these.

1. Draw each of the following prisms in the space provided. Make sure you use a ruler. The front and back faces have been drawn for you.

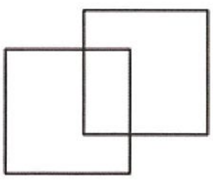
    a) Triangular prism.

    b) Hexagonal prism

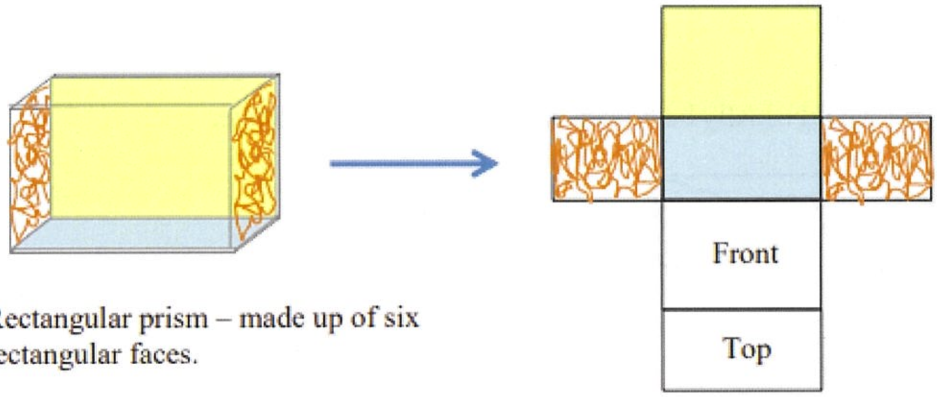
    

    c) Cube

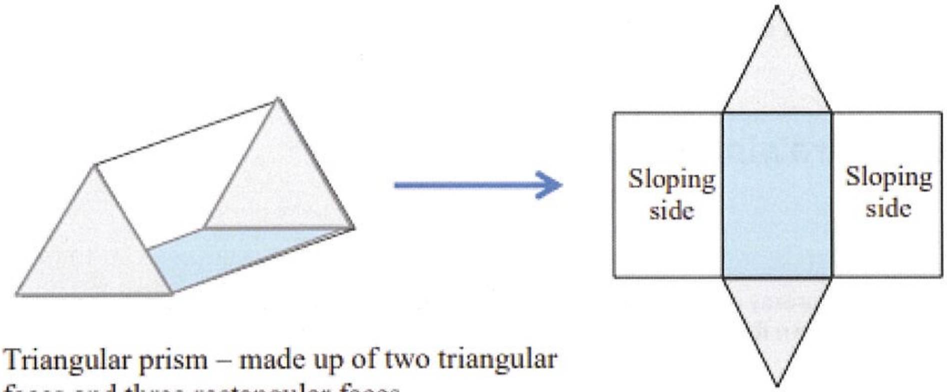
    

Nets of prisms

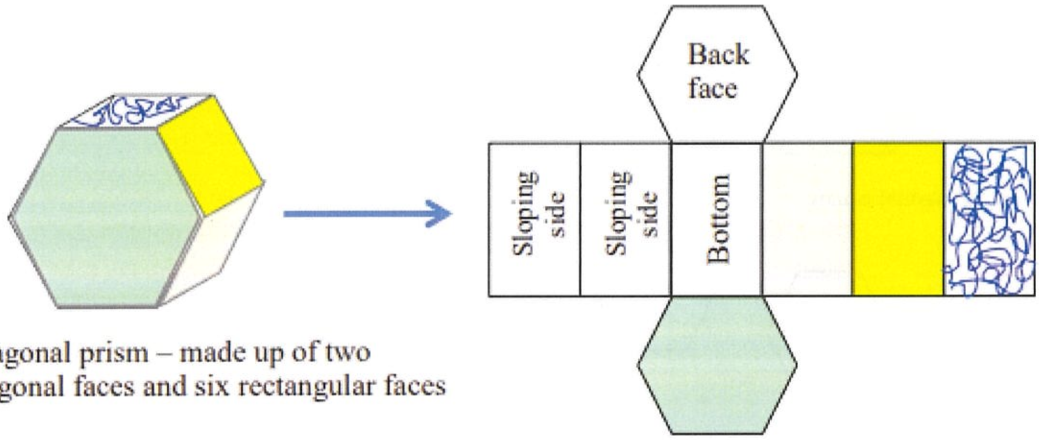
The net of a solid is the plane (or flat) figure that is formed when we 'unfold' the solid.



Rectangular prism – made up of six rectangular faces.



Triangular prism – made up of two triangular faces and three rectangular faces.



Hexagonal prism – made up of two hexagonal faces and six rectangular faces

As seen in the examples above, the simplest way to draw the net of a prism is to imagine unfolding all the panels that wrap around the shape and then flipping out the front and back face.

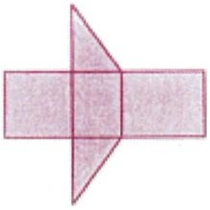
After drawing a net, always check that the number of faces you have drawn match the number of faces in the original solid.

Activity – Nets of prisms

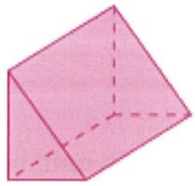
Try these.

1. Match each net in parts i) to iv) with one of the solids shown in parts A to D below.

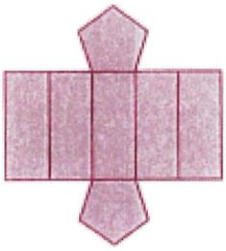
i)



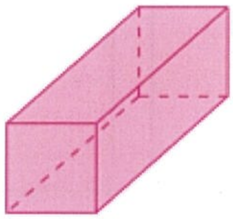
A



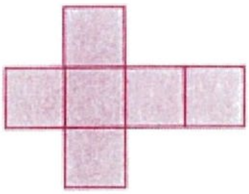
ii)



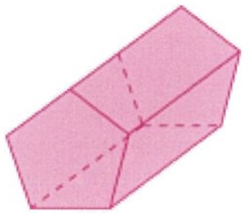
B



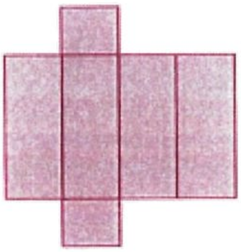
iii)



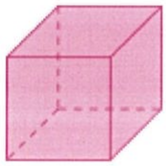
C



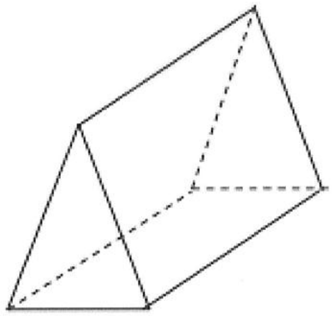
iv)



D

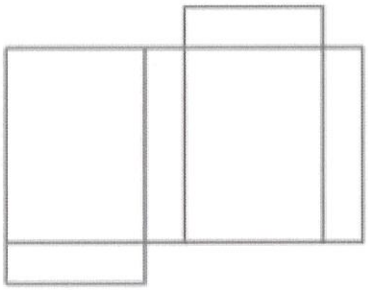


2. Using a ruler and a pen, draw the net of the following triangular prism in the space provided.

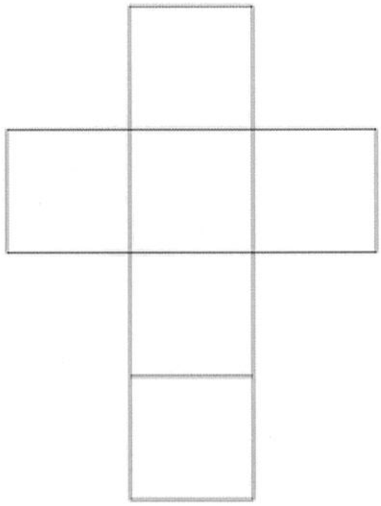


3. Name the prism that would be formed from the following nets.

a)



b)



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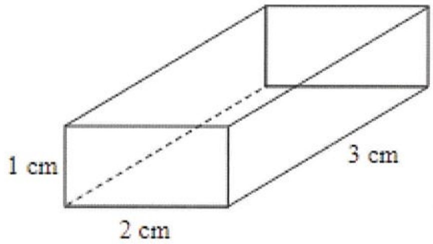
Surface area using nets

The surface area of a prism is the total area of its faces.

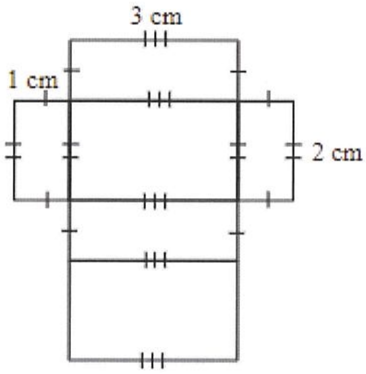
To find the surface area of any solid, you need to first find the area of each face and add all these areas together. Finding the surface area using the net can be useful as it allows you to see each of the different faces that need to have their areas calculated.

Rectangular prisms

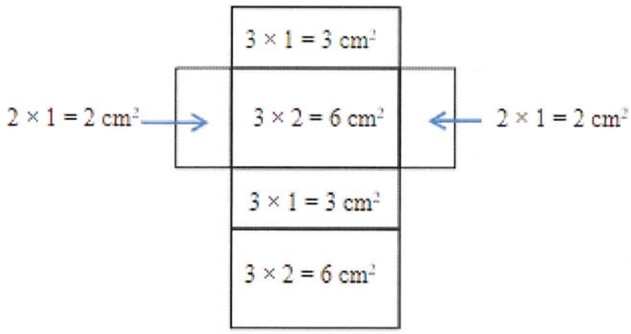
To find the surface area of the rectangular prism below we could draw the net and find the area of each of the faces.



This rectangular prism is made up of 6 faces, all of which are rectangles. Another important thing to note is that each pair of opposite faces are identical.



Note that the symbols used indicate where the lengths are the same. So all lines marked with three dashes for instance are 3 cm long for this example.



Adding up each of the areas of the faces:

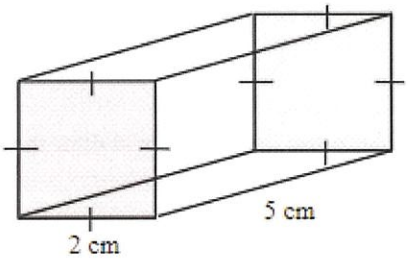
$$\begin{aligned}\text{Total area} &= 3 + 6 + 3 + 6 + 2 + 2 \\ &= 22 \text{ cm}^2\end{aligned}$$

Therefore the surface area of the prism is 22 cm^2.

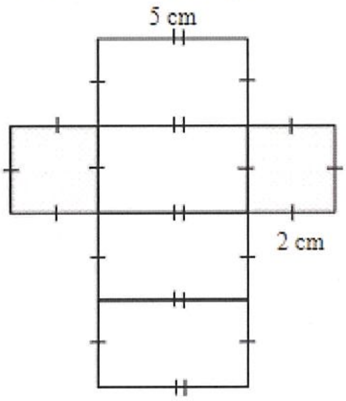
Square prisms

A square prism is like a rectangular prism only it has one pair of identical faces that are squares. The other four faces are identical rectangles.

An example of a square prism is shown below.



The net of the square prism could look like this:



The square faces each have an area of $2 \times 2 = 4$ cm^2

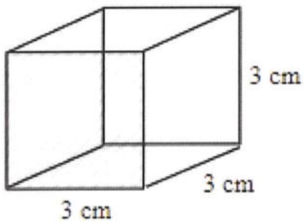
The rectangular faces each have an area of $5 \times 2 = 10$ cm^2

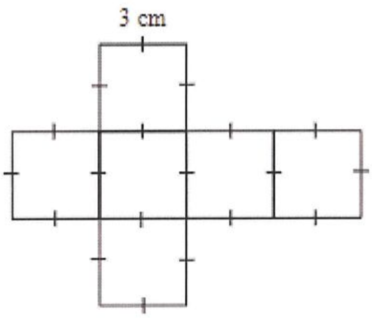
So total surface area = 2 × square area + 4 × rectangular area
$= 2 \times 4 + 4 \times 10$
$= 8 + 40$
$= 48$ cm^2

Cubes

Cubes are special types of rectangular prisms, where every face is a square with the same dimensions and the same area.

To find the surface area of this cube, which has a side length of 3 cm, we could draw the net.





The area of one square $= 3 \times 3$
$= 9$ cm^2

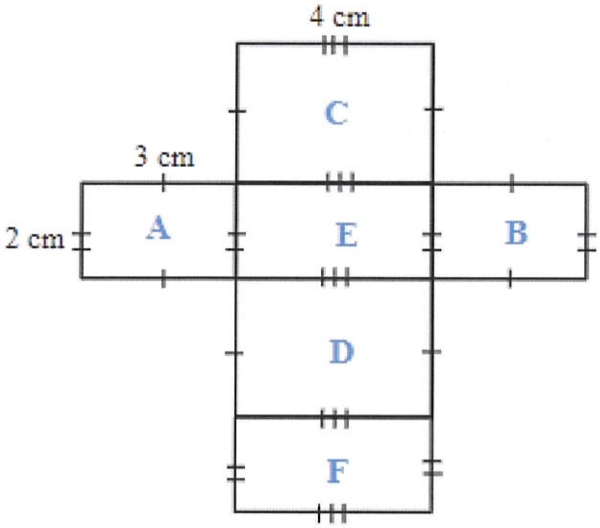
As each of the six faces are identical, total surface area $= 6 \times 9$
$= 54$ cm^2

Therefore the surface area of this cube is 54 cm^2.

Note: this could be done in one calculation, as $6 \times (3 \times 3) = 54$ cm^2.

Ex C.1

1. Refer to the net below to answer the following questions



a) What solid shape does this net represent?

____

b) Find the area of i) rectangle A _______________________________

____

ii) rectangle B_______________________________

____

iii) rectangle C_______________________________

____

iv) rectangle D _______________________________

____

v) rectangle E_______________________________

____

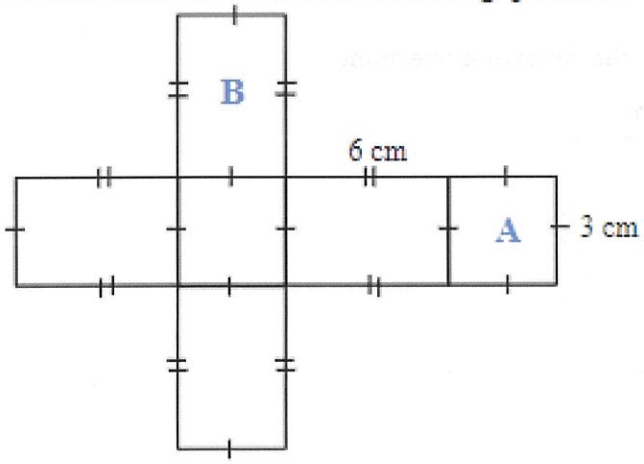
vi) rectangle F _______________________________

____

c) Calculate the total surface area of the prism _______________________

____

2. Refer to the net below to answer the following questions.



a) What solid shape does this net represent?

____

b) Find the area of i) shape A ______________________________

____

ii) shape B ______________________________

____

c) Calculate the total surface area of the prism.

____

____

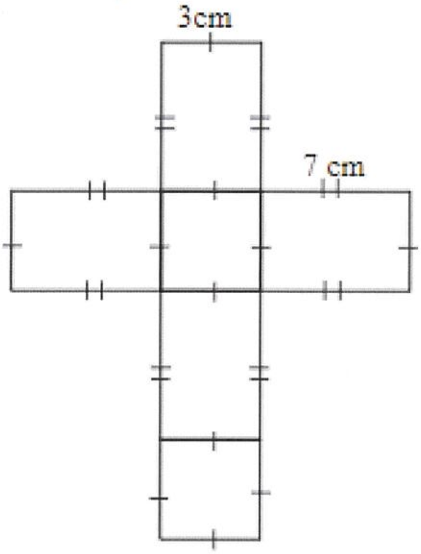
____

____

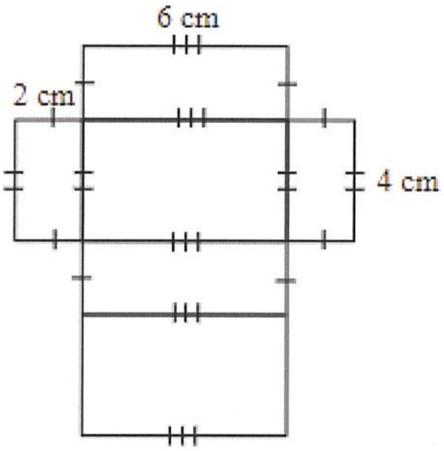
____

3. Calculate the surface area of the following prism that would be formed using the nets given. Note that they are not drawn to scale.

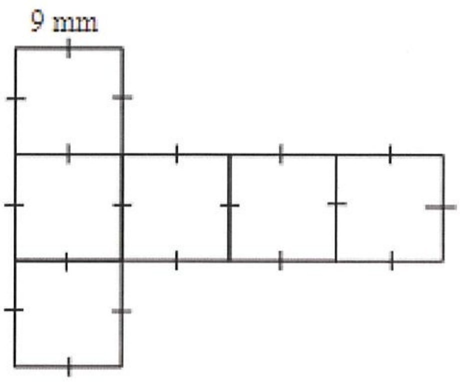
a)



b)

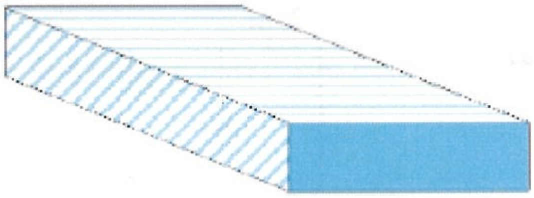


c)



Ex C.2

1. Answer the following questions about this prism.



This rectangular prism is 10 mm high. The top is 45 mm by 60 mm.

a) Write the three dimensions in the correct place on the prism.

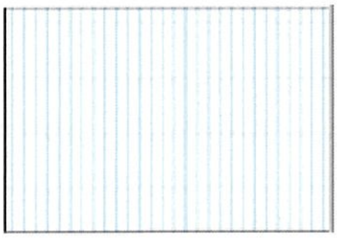
b) Write the correct length and breadth on the sides of these rectangles below. These three rectangles are the same shape as the similarly shaded faces of the prism.

i)  ______

______

ii)  ______

______

iii)  ______

______

c) Calculate the area of these three rectangles.

i) ____

____

ii) ____

____

iii) ____

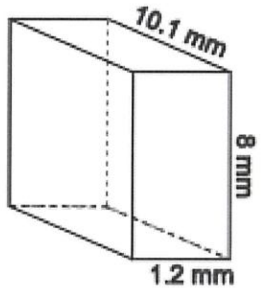
____

d) Calculate the total surface area of this prism.

____

____

2. Calculate the total surface area of this prism. Write your answer correct to one decimal place.



____

____

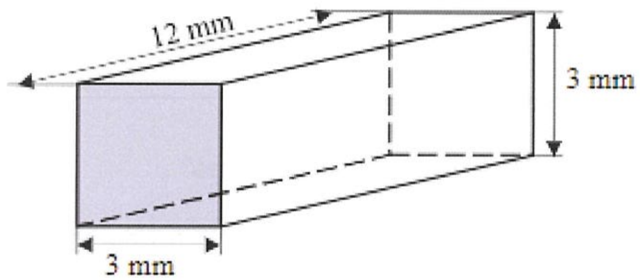
____

____

____

____

3. Use the prism shown to answer the following questions.



a) What shape is the shaded face? ______________________________

b) Complete the sentence: This solid is a special type of rectangular prism.

It is called a ____________________ prism.

c) Calculate the area of the square face.

____

____

d) Calculate the area of the top face.

____

____

e) How many faces have the same area as the top? ____________________

f) Calculate the total surface area of the prism.

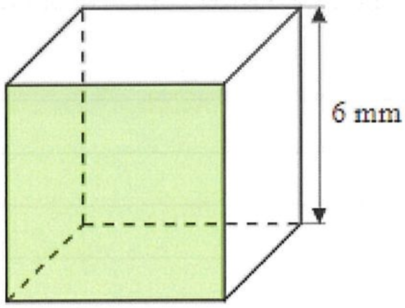
____

____

____

____

4. a) Calculate the area of the front face of this cube.



____

____

b) What is the total surface area?

____

____

5. The area of one face of a cube is 49 cm^2. What is its total surface area?

____

____

6. The edge of a cube is 12 mm long.

a) Draw a neat sketch of the cube in the space below.

b) Use your calculator to find the total surface area.

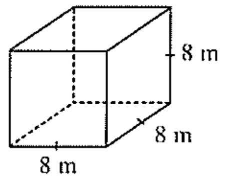
____

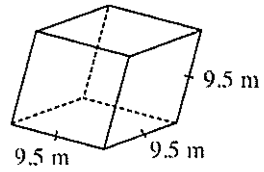
____

____

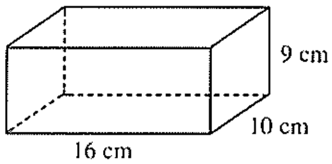
Ex C.3

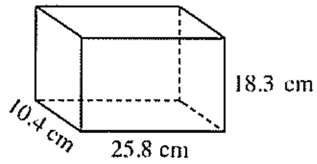
Question 1 Find the surface area of each cube.

a


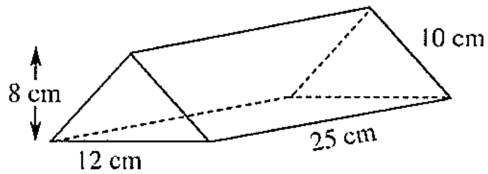
b


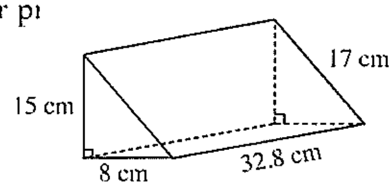
Question 2 Find the surface area of each rectangular prism.

a


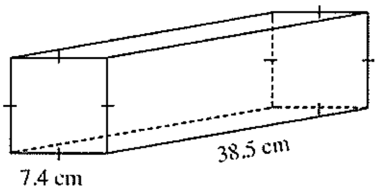
b


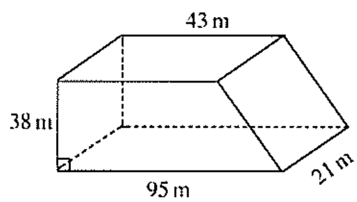
Question 3 Find the surface area of each triangular pı

a


b


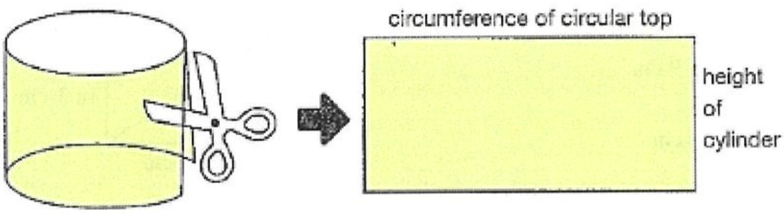
Question 4 Find the surface area of each shape.

a


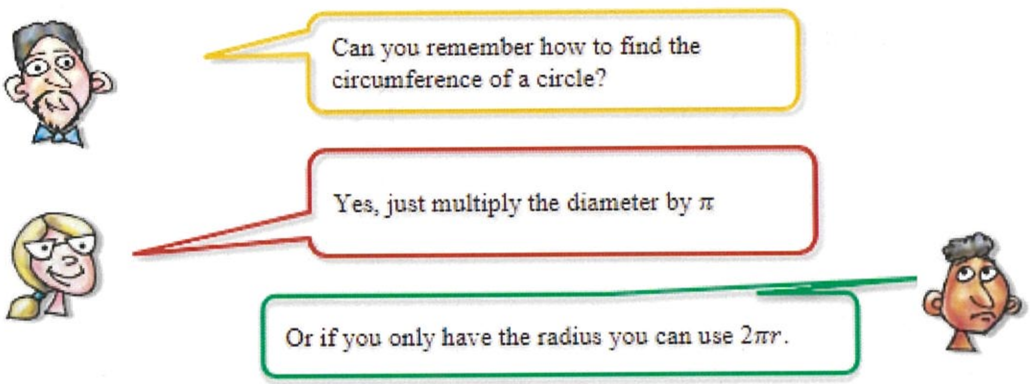
b


The net of a cylinder

If you cut the curved part of an open cylinder, parallel to its perpendicular height, you get a rectangle.



The length of this rectangle is the same as the circumference of the circular ends of the cylinder and its width is the same as the cylinder's height.



You can write the area of this rectangle in different ways.

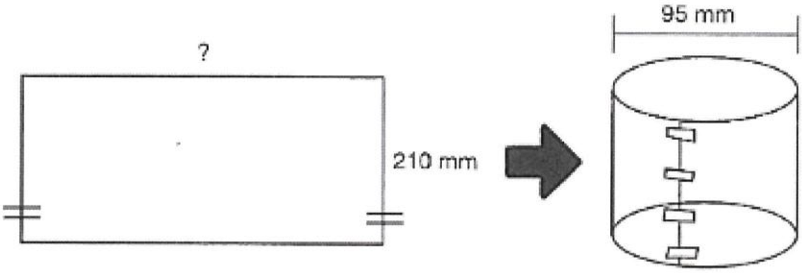
- $C \times h$ where $C = 2\pi r$
- πdh, because $C = \pi \times$diameter
- $2\pi rh$, because $d = 2r$

Use the formula you prefer in the following activity.

Activity – The net of a cylinder

Try these.

1. The opposite side of a rectangular piece of paper are sticky-taped together to form an open cylinder. These sides are both 210 mm long.



a) If the diameter of this newly formed cylinder is 95 mm, how long is the piece of paper?

____

____

____

b) Calculate the area of paper used to make this open cylinder. Write your answer correct to two significant figures.

____

____

____

NB: Don't look at answer (upside down) until you have had a go!

1. a) about 298 mm.

The unknown side = circumference of circular end

$C = \pi d$

$= \pi \times 95$

≈ 298 mm

(Your calculator display should read 298.45 …)

b) 63 000 mm^2

Area of rectangle (or curved side) $= C \times h$

$\approx 298 \times 210$

$\approx 63\,000$ mm^2

Note: you should use the answer in your calculator display, from above, rather than the approximate answer of 298. (If the questions had asked for three significant figures, your answer would be different if you used 298.

Activity – The net of a cylinder

Try these.

3. The height of a closed cylinder is 25 cm and its diameter is the same as its height.

   a) Complete the following
   SA = ? r = ______________________ h = ______________________

   b) Substitute these values into the correct place in the formula below.

   $SA = 2\pi r^2 + 2\pi rh$

   ____

   ____

   b) Use your calculator to find the surface area of this cylinder, then write your answer correct to three significant figures.

   ____

   ____

NB: Don't look at answer (upside down) until you have had a go!

Suggested Answers:

3. a) SA = ? $r = 12.5$ cm (half of 25 cm,) $h = 25$ cm

   b) $SA = 2\pi r^2 + 2\pi rh$

   $= 2\times\pi\times 12.5^2 + 2\times\pi\times 12.5\times 25$

   c) $= 2950\text{ cm}^2$

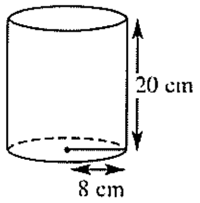
   You could work from left to right using your calculator like this:

   2 × π × 12.5 x^2 + 2 × π × 12.5 × 25 =

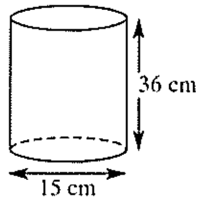
   Your calculator display should be: 2945.2 …

   or you could calculate this surface area, piece by piece, by first finding the area of one circular end, then doubling this answer. You then need to store this answer in your calculator memory. After this you must calculate the area of the curved surface (remembering to use the circumference formula.) this answer then needs to be added to your first answer which is in your calculator memory.

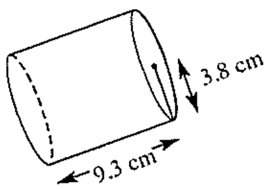
QUESTION 1 For each cylinder, find the following correct to two decimal places.
i the area of a circular base **ii** the area of the curved surface



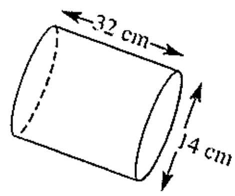
b



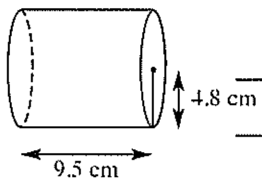
QUESTION 2 Find the curved surface area of each cylinder in terms of π.



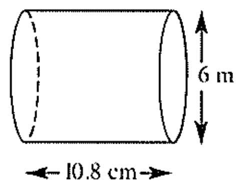
b



QUESTION 3 For each cylinder, find the following correct to three significant figures.
i the combined area of the two circular ends **ii** the area of the curved surface
iii the total surface area



b

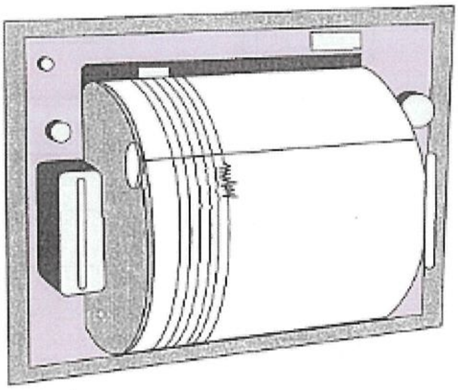


QUESTION 4 Find the total surface area of the outside of a pipe 20 m long with an outer radius 0.75 m (a pipe does not have any ends). Give your answer correct to one decimal place.

Activity – Calculating other dimensions

Try these.

1. The helicorder drum below is used to record seismic activity in the Earth's interior.



The paper that is wrapped around its curved surface has an area of 3580 cm^2. If this cylindrical drum is 40 cm wide, calculate its radius.

2. The surface area of a cylinder is 400 m^2.

   a) If the height and radius of this cylinder are the same, use $r = h$ to show how the formula, $SA = 2\pi r^2 + 2\pi rh$ changes to $SA = 4\pi h^2$.

   b) Use this formula to calculate the value of h^2.

   c) Use the square root key, $\sqrt{\square}$, on your calculator to find the height of this cylinder correct to one decimal place.

NB: Don't look at answer (next page) until you have had a go!

Suggested Answers:

1. 14.2 cm

There are two ways you can do this, although they are essentially the same.

Method 1: use the curved surface area formula and substitute the given values to find r.

Curved surface area = 3580 cm^2, r = ?, h = 40 cm.

$$
\begin{aligned}
\text{Curved surface area} &= 2\pi rh \\
3580 &= 2\times\pi\times r\times 40 \\
3580 &= 80\pi\times r \\
3580 \div 80\pi &= r \\
r &\approx 14.2
\end{aligned}
$$

Don't forget that you can calculate $3580 \div 80\pi$ by using continuous division or by using brackets. You might use these keys:

- 3580 ÷ 80 ÷ π = or
- 3580 ÷ (80 × π) =

Method 2: first find the length of the paper, then, as this is the same as the circumference of the cylinder, use $C = 2\pi r$ (or $C = \pi d$) to calculate the radius of the cylinder.

$$
\begin{aligned}
\text{Length of paper} &= \text{area of rectangle} \div \text{width} \\
&= 3580 \div \text{width} \\
&= 89.5 \text{ cm}
\end{aligned}
$$

$C = 89.5$, r =?

$$
\begin{aligned}
C &= 2\pi r \\
89.5 &= 2\pi\times r \\
89.5 \div 2\pi &= r \\
r &\approx 14.2 \text{ cm}
\end{aligned}
$$

2. a) if $r = h$ then this means that you must replace r with h.

$$
\begin{aligned}
SA &= 2\pi r^2 + 2\pi rh \\
SA &= 2\pi h^2 + 2\pi h^2 \\
SA &= 4\pi h^2
\end{aligned}
$$

b) about 31.8 m

Now replace SA with 400 and solve for h^2

$$
\begin{aligned}
SA &= 4\pi h^2 \\
400 &= 4\pi h^2 \\
100 &= \pi h^2 \text{ (By dividing both sides by 4.)} \\
h^2 &= \frac{100}{\pi} \\
&\approx 31.830.....
\end{aligned}
$$

c) about 5.6 m

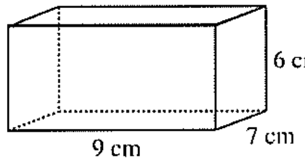
The best method is to simply press the square root key (followed by the equals sign for some calculators,) while you still have the value of h^2 in your calculator display. Your sequence of keys might look like this

100 ÷ π = √ =

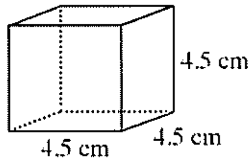
Ex C.5

QUESTION 1 Find the surface area of the following rectangular prisms.

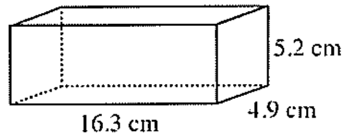
a



b

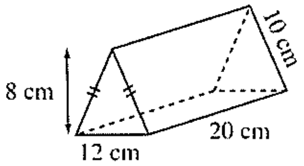


c

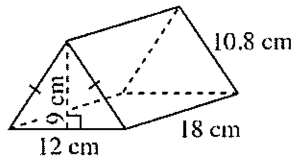


QUESTION 2 Find the surface area of the following triangular prisms.

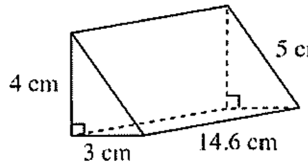
a



b

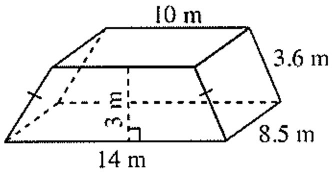


c

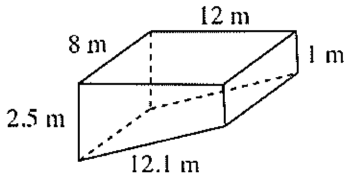


QUESTION 3 Find the surface area of the following trapezoidal prisms.

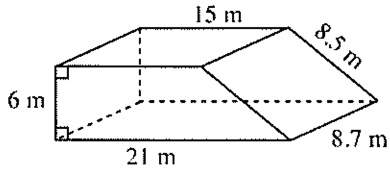
a



b

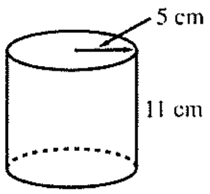


c

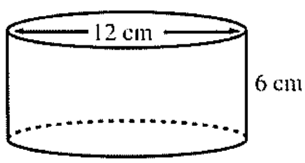


QUESTION 4 Find the surface area of the following solid cylinders.

a



b



c